

Principles of Artificial Intelligence

Problem Solving as Search

Outline

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- 2 [Example Problems](#)
- 3 Search [Generalities](#)
- 4 [Uninformed Search Strategies](#)
 - Breadth-First [Search](#)
 - Uniform-cost [Search](#)
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 - [Depth-Limited Search & Iterative Deepening](#)
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 - Greedy [Search](#)
 - [A* Search](#) Heuristic Functions

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 - Uniform-cost Search
 - Depth-First Search
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Problem Solving as Search

One of the dominant approaches to AI problem solving: **formulate a problem/task as search in a state space**.

- 1. **Goal formulation:** define the successful world states
 - Ex: a set of states, a Boolean test function ...
- 2. **Problem formulation:**
 - define a representation for states
 - define legal actions and transition functions
- 3. **Search:** find a solution by means of a search process
 - solutions are sequences of actions

Execution: given the solution, perform the actions

⇒ **Problem-solving agents** are (a kind of) **goal-based agents**

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Problem Solving as Search: Example

Example: Traveling in Romania

- **Informal description:** On holiday in Romania; currently in Arad. Flight leaves tomorrow from Bucharest
- **Formulate goal:** (Be in) Bucharest
- **Formulate problem:**
 - States: various cities
 - Actions: drive between cities
 - Initial state: Arad
- **Search for a solution:**
sequence of cities from Arad
to Bucharest
e.g. Arad, Sibiu, Fagaras, Bucharest
explore a search tree/graph

Note

The agent is assumed to have no heuristic knowledge about traveling in Romania to exploit

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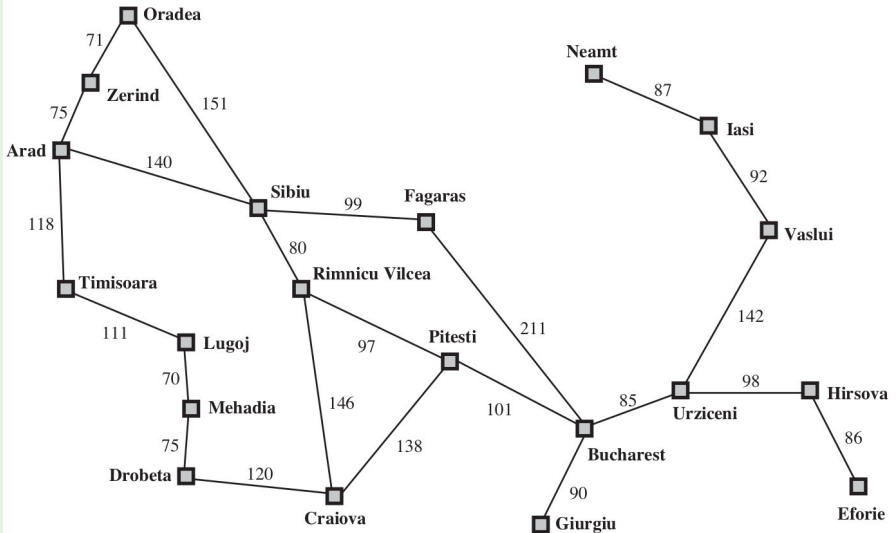
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Problem Solving as Search: Example [cont.]

A simplified road map of part of Romania.



Problem Solving as Search [cont.]

Assumptions for Problem-solving Agents (this chapter

- only) state representations are **atomic**

⇒ world states are considered as wholes, with no internal structure

- Ex: Arad, Sibiu, Zerind, Bucharest,...

the environment is **observable**

⇒ the agent always knows the current state

- Ex: Romanian cities & roads have signs

the environment is **discrete**

⇒ at any state there are only finitely many actions to choose from

- Ex: from Arad, (go to) Sibiu, or Zerind, or Timisoara (see map)

the environment is **known**

⇒ the agent knows which states are reached by each action

- ex: the agent has the map

the environment is **deterministic**

⇒ each action has exactly one outcome

Ex: from Arad choose go to Sibiu ⇒ next step in Sibiu

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Notes about search

- Search happens **inside the agent**
 - a **planning** stage before acting different from **searching in the world**
- An agent is given **a description of what to achieve**, not an **algorithm**
⇒ the only possibility is to **search for a solution**
Searching can be computationally very demanding (NP-hard)
- Can be driven with benefits by knowledge of the problem (heuristics)
⇒ **informed/heuristic search**

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Problem-solving Agent: Schema

function SIMPLE-PROBLEM-SOLVING-AGENT(*percept*) **returns** an action

persistent: *seq*, an action sequence, initially empty

state, some description of the current world state

goal, a goal, initially null

problem, a problem formulation

state \leftarrow UPDATE-STATE(*state*, *percept*)

if *seq* is empty **then**

goal \leftarrow FORMULATE-GOAL(*state*)

problem \leftarrow FORMULATE-PROBLEM(*state*, *goal*)

seq \leftarrow SEARCH(*problem*)

if *seq* = *failure* **then return** a null action

action \leftarrow FIRST(*seq*)

seq \leftarrow REST(*seq*)

return *action*

(Oc S. Russell & P. Norwig, AIMA)

While executing the solution sequence the agent ignores its percepts when choosing an action since it knows in advance what they will be (they will be system")

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  if seq is empty then
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While executing the solution sequence the agent ignores its percepts when choosing an action since it knows in advance what they will be (open-loop system)

Well-defined problems and solutions

Problem Formulation: Components

- the **initial state** the agent starts in
 - Ex: $In(Arad)$

the set of **applicable actions** available in a state ($ACTIONS(s)$)

- Ex: if s is $In(Arad)$, then the
 - applicable actions are
 - $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$

a description of what each action does (aka **transition**

- model**) $RESULT(s,A)$: state resulting from applying action A in state s Ex: $RESULT(IN(ARAD), GO(ZERIND))$ is $IN(ZERIND)$

the **goal test** determining if a given state is a goal state

Explicit (e.g.: $\{In(Bucharest)\}$) Implicit
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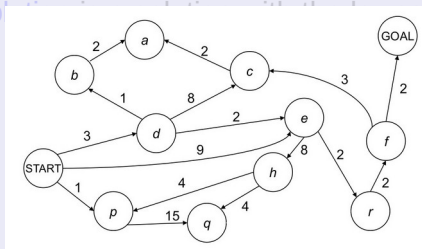
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Well-defined problems and solutions [cont.]

State Space, Graphs, Paths, Solutions and Optimal Solutions

Initial state, **actions**, and **transition model** implicitly define the **state space** of the problem

- the state space forms a **directed graph** (e.g. the Romania map)
 - typically too big to be created explicitly and be stored in full
- in a state space graph, each state occurs only once
- a **path** is a sequence of states connected by actions
- a **solution** is a path from the initial state to a goal state
- an **optimal solution** is a path from the initial state to a goal state with the lowest cost

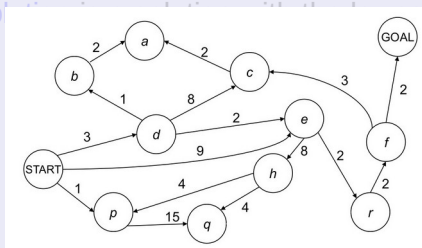


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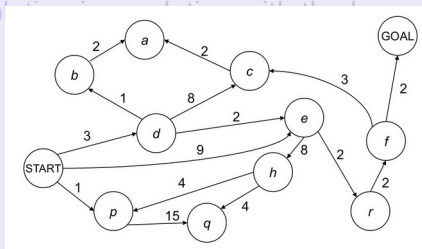


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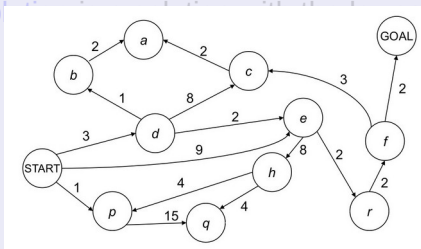
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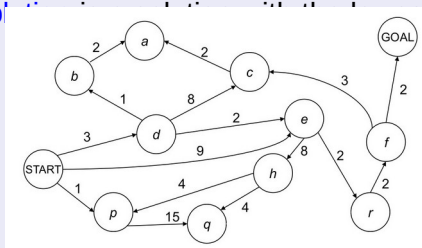


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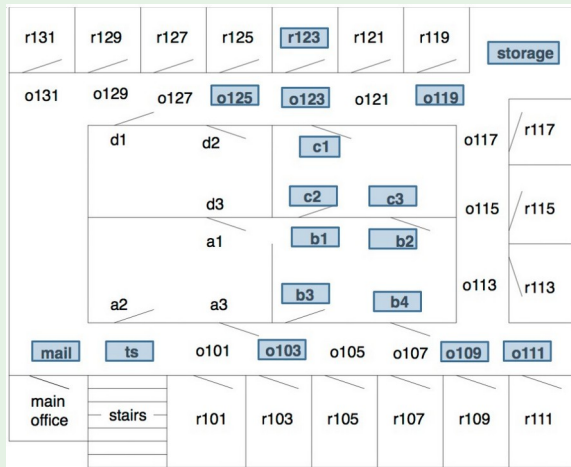
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Example: Path finding for a Delivery Robot

Task: move from o103 to r123

- States
- Initial state
- Goal state
- State graph
- Optimal solution

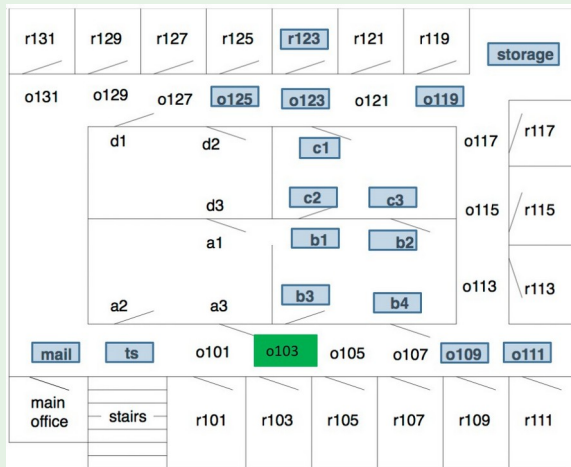


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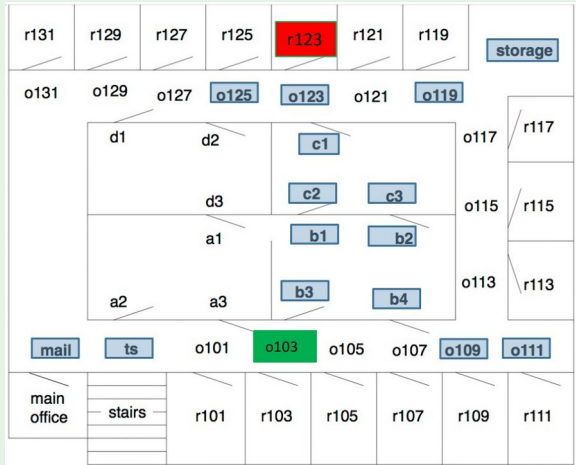


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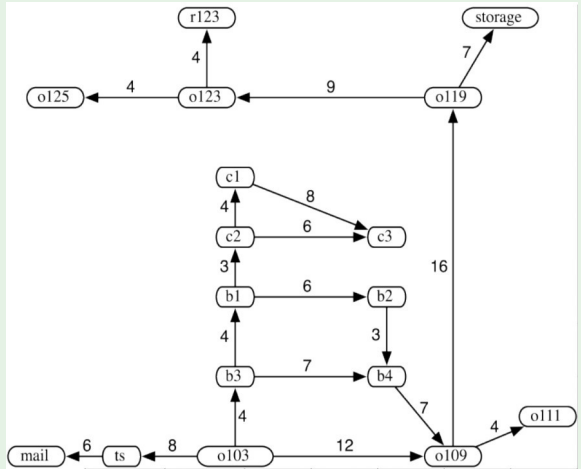


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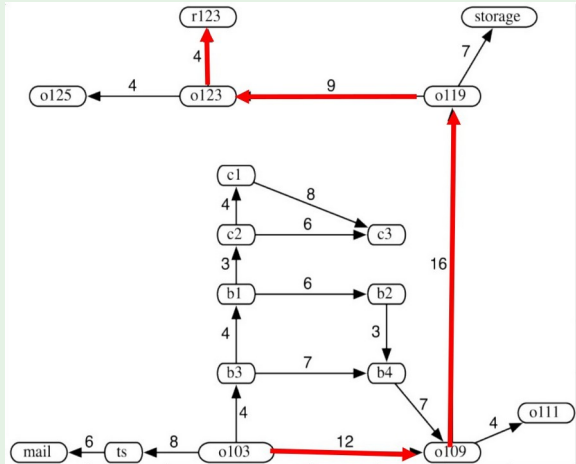


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Abstraction

Problem formulations are **models** of reality (i.e. **abstract**

- **descriptions**) real world is absurdly complex
⇒ state space must be **abstracted** for problem solving
lots of details removed because irrelevant to the problem
 - Ex: exact position, “turn steering wheel to the left by 20 degree”, ...
- **abstraction**: the process of removing detail from representations
 - abstract state** represents many real states
 - abstract action** represents complex combination of real actions
- valid abstraction**: can expand any abstract solution into a solution in the detailed world
- useful abstraction**: if carrying out each of the actions in the solution is easier than in the original problem

The **choice of a good abstraction** involves **removing as much detail as possible**, while **retaining validity** and **ensuring that the abstract actions are easy to carry out**.

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Well-defined problems and solutions [cont.]

Abstraction

Problem formulations are **models** of reality (i.e. **abstract**

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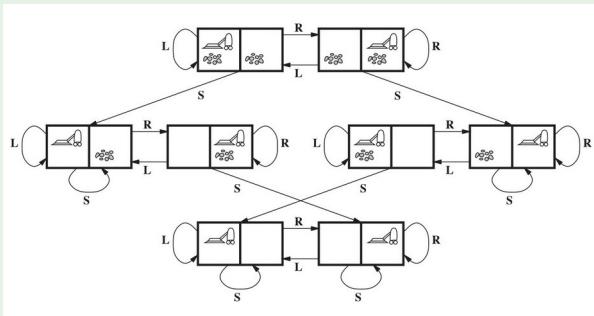
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Outline

- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
- 4 Uninformed Search Strategies
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 Informed Search Strategies
 - Greedy Search
 - A* Search Heuristic Functions

Toy Example: Simple Vacuum Cleaner

- **States:** 2 locations, each $\{clean, dirty\}$: $2 \cdot 2^2 = 8$
- **Initial State:** any
- **Actions:** $\{Left, Right, Suck\}$
- **Transition Model:** Left [Right] if A [B], Suck if clean \Rightarrow no effect
- **Goal Test:** check if squares are clean
- **Path Cost:** each step costs 1 \Rightarrow path cost is # of steps in path



Toy Example: The 8-Puzzle

- **States:** Integer location of each tile \Rightarrow **9!/2 reachable**
- **Initial State:** any
- **Actions:** moving $\{Left, Right, Up, Down\}$ the empty space
- **Transition Model:** empty switched with the tile in target location
- **Goal Test:** checks state corresponds with goal configuration
- **Path Cost:** each step costs 1 \Rightarrow path cost is # of steps in path

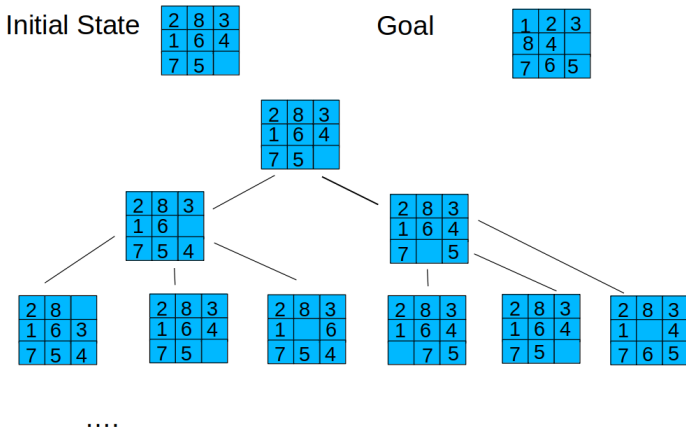
7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

Toy Example: The 8-Puzzle [cont.]



(Courtesy of Michela Milano, UniBO)

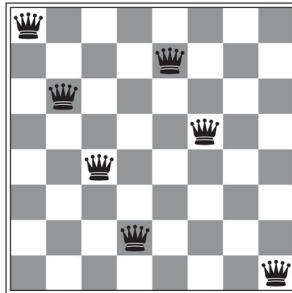
NP-complete: N-Puzzle ($N = k^2 - 1$): $N!/2$ reachable states

Toy Example: 8-Queens Problem

- **States:** any arrangement of 0 to 8 queens on the board
 $\Rightarrow 64 \cdot 63 \cdot \dots \cdot 57 \approx 1.8 \cdot 10^{14}$ possible sequences
- **Initial State:** no queens on the board
- **Actions:** add a queen to any empty square
- **Transition Model:** returns the board with a queen added

Goal Test: 8 queens on the board
Path Cost:

not attacked by other

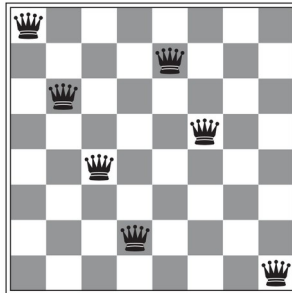


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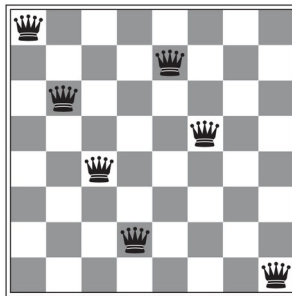
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Toy Example: 8-Queens Problem (incremental)

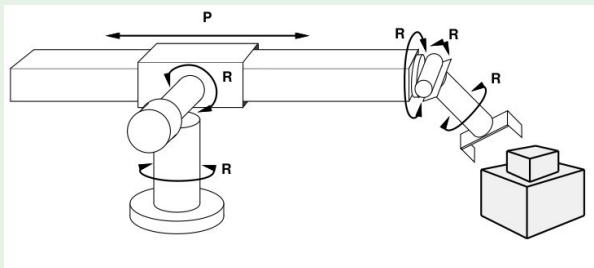
- **States:** $n \leq 8$ queens on board, one per column in the n leftmost columns, **no queen attacking another**
 \Rightarrow 2057 possible sequences
- **Actions:** Add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.

...



Real-World Example: Robotic Assembly

- **States:** real-valued coordinates of robot joint angles, and of parts of the object to be assembled
- **Initial State:** any arm position and object configuration
- **Actions:** continuous motions of robot joints
- **Transition Model:** position resulting from motion
- **Goal Test:** complete assembly (without robot)
- **Path Cost:** time to execute



Other Real-World Examples

- Airline travel planning problems
- Touring problems
- VLSI layout problem
- Robot navigation
- Automatic assembly sequencing
- Protein design
- ...

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Searching for Solutions

Search: Generate sequences of actions.

- **Expansion:** one starts from a state, and applying the operators (or successor function) will generate new states
- **Search strategy:** at each step, choose which state to expand.
- **Search Tree:** It represents the expansion of all states starting from the initial state (the **root** of the tree)
- The **leaves** of the tree represent either:
 - states to expand
 - solutions
 - dead-ends

Tree Search Algorithms

Tree Search: Basic idea

- Off-line, simulated exploration of state space
 - start from initial state
 - pick one leaf node, and generate its successors (a.k.a. **expanding** a node) states)
 - set of current leaves called **frontier** (a.k.a. **fringe**, **open list**) strategy for picking leaves critical (**search strategy**)
 - ends when either **a goal state is reached**, or **no more candidates to expand are available** (or time-out/memory-out occur)

function TREE-SEARCH(*problem*) **returns** a solution, or failure

initialize the frontier using the initial state of *problem*

loop do

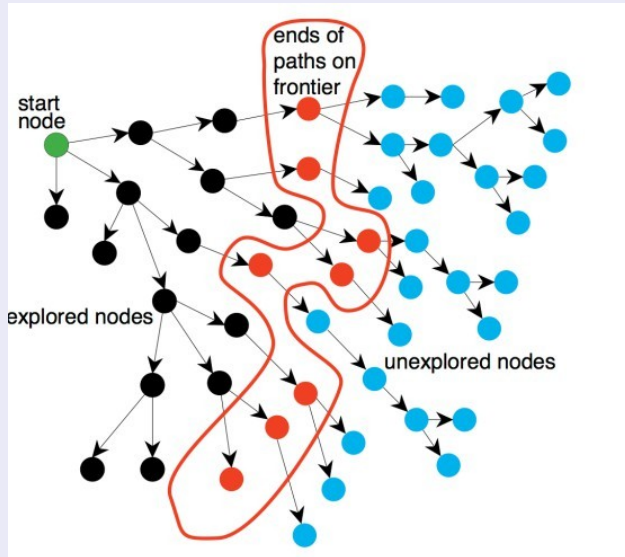
if the frontier is empty **then return** failure

 choose a leaf node and remove it from the frontier

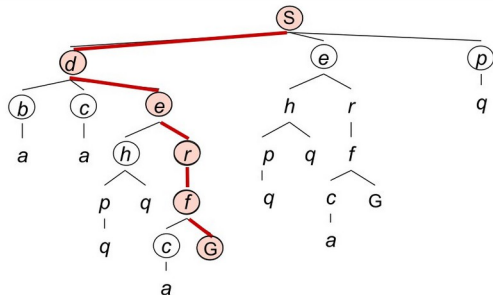
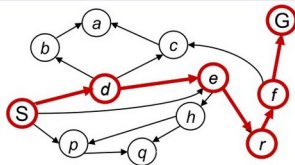
if the node contains a goal state **then return** the corresponding solution

 expand the chosen node, adding the resulting nodes to the frontier

Tree Search Algorithms [cont.]



Tree-Search Example

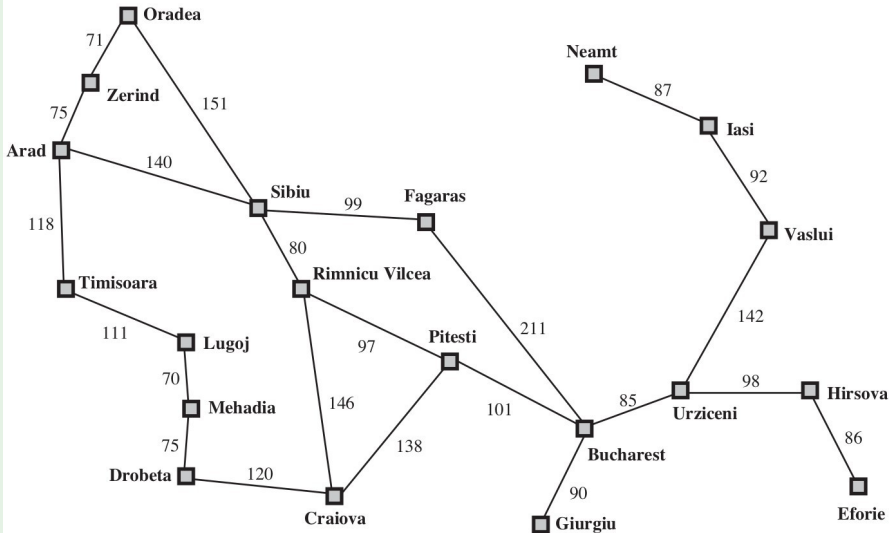


~~S~~
~~S → d~~
 S → e
 S → p
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 S → d → c
~~S → d → e~~
 S → d → e → h
~~S → d → e → r~~
~~S → d → e → r → f~~
 S → d → e → r → f → c
~~S → d → e → r → f → G~~

(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

Tree-Search Example: Trip from Arad to Bucharest

A simplified road map of part of Romania.



Tree-Search Example: Trip from Arad to Bucharest

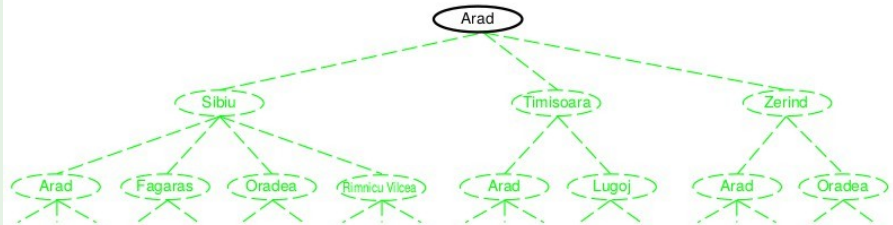
Expanding the search tree

- Initial state: {Arad}

Expand initial state \Rightarrow

- {Sibiu, Timisoara, Zerind}
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Pick&expand Sibiu \Rightarrow



(Oc S. Russell & P. Norwig, AIMA)

Beware: Arad \rightarrow Sibiu \rightarrow Arad (repeated state \Rightarrow loop)

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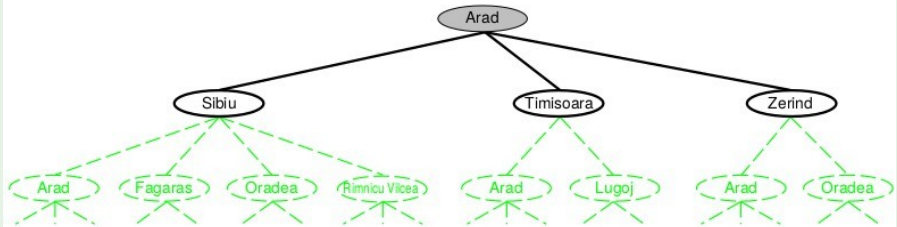
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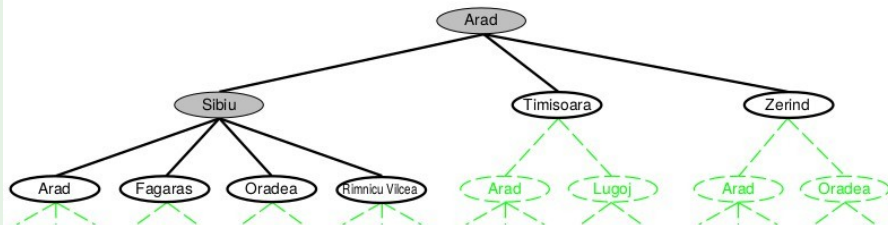
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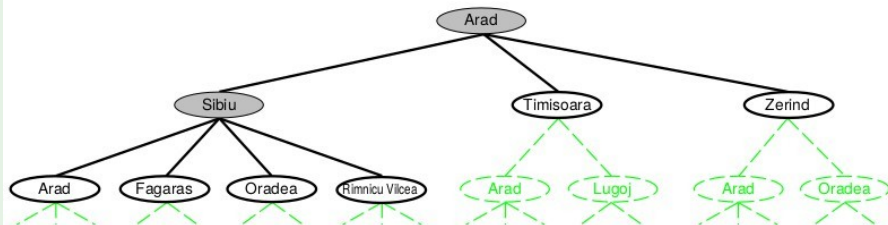
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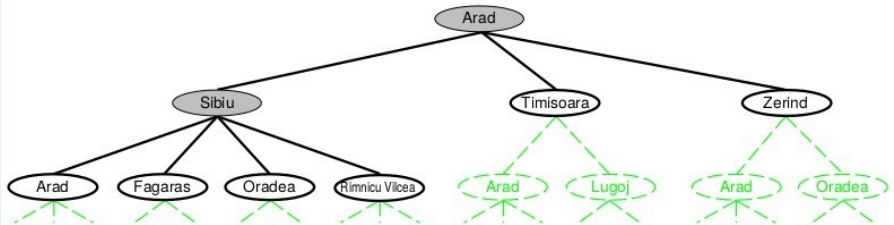
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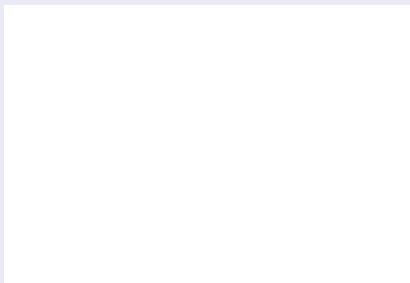


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Repeated states & Redundant Paths

- **redundant paths** occur when there is more than one way to get from one state to another
⇒ same state explored more than once
- Failure to detect repeated states can:
 - cause infinite loops
 - turn linear problem into exponential

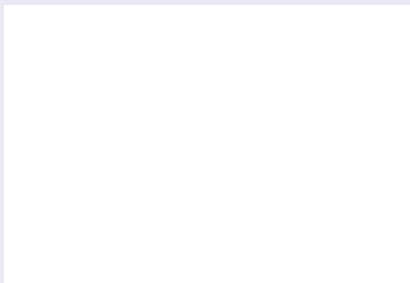


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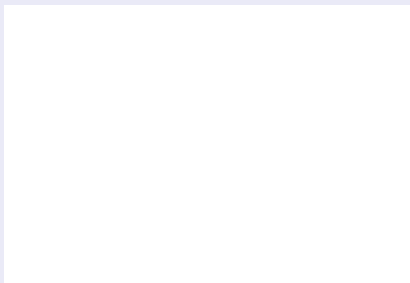


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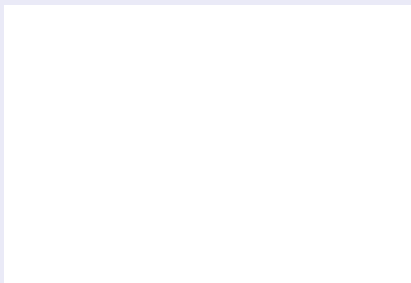


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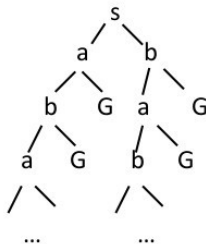
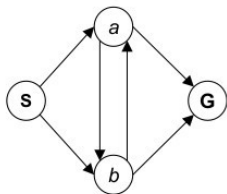


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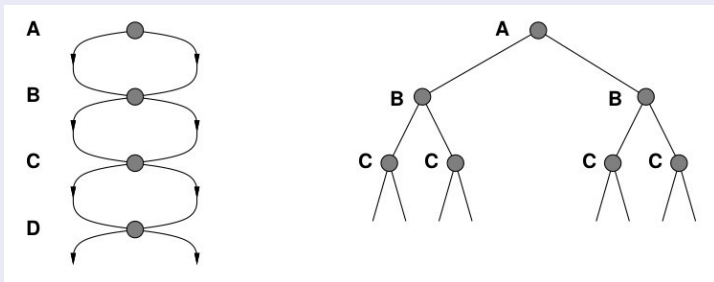


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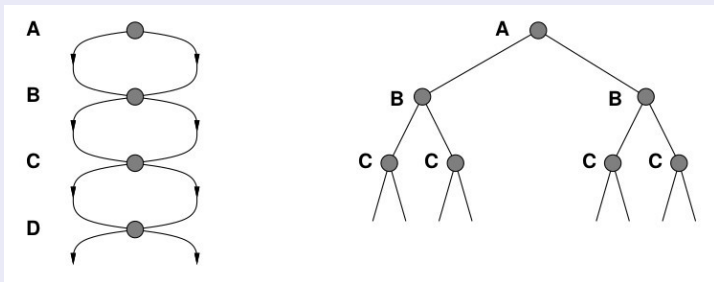


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Graph Search Algorithms

Graph Search: Basic idea

- add a data structure which remembers every expanded node
 - a.k.a. explored set or closed list
 - typically a hash table (access $O(1)$)
- do not expand a node if it already

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure

initialize the frontier using the initial state of *problem*

initialize the explored set to be empty

loop do

if the frontier is empty **then return** failure

choose a leaf node and remove it from the frontier

if the node contains a goal state **then return** the corresponding solution

add the node to the explored set

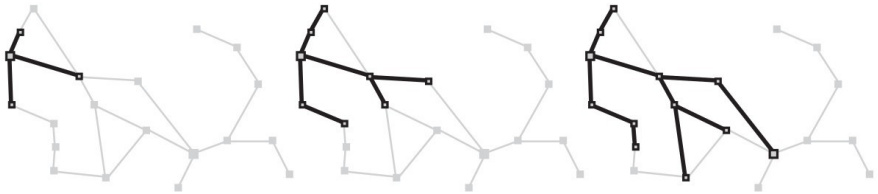
expand the chosen node, adding the resulting nodes to the frontier

only if not in the frontier or explored set

Graph Search Algorithms: Example

Graph search on the Romania trip problem

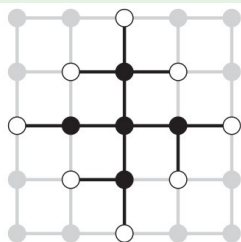
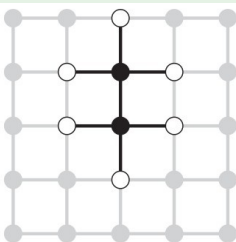
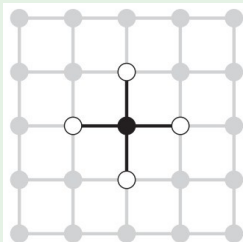
- (at each stage each path extended by one
- step) two states become dead-end



(Oc S. Russell & P. Norwig, AIMA)

Graph Search Algorithms: Example

Separation Property of graph search: the frontier separates the state-space graph into the **explored region** and the **unexplored region**



(Oc S. Russell & P. Norwig, AIMA)

Implementation: States vs. Nodes

- A **state** is a representation of a physical configuration
- A **node** is a data structure constituting part of a search tree

includes fields: state, parent, action, path cost $g(x)$

⇒ node \neq state

It is easy to compute a child node from its parent

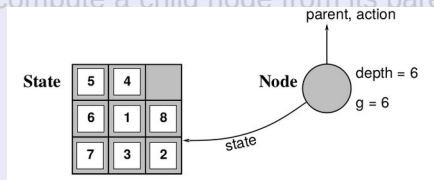
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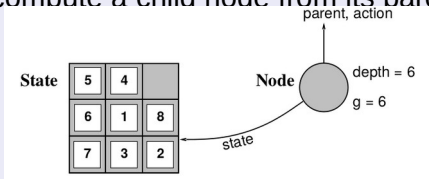
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function CHILD-NODE(*problem*, *parent*, *action*) **returns** a node
return a node with

STATE = *problem*.RESULT(*parent*.STATE, *action*),

PARENT = *parent*, ACTION = *action*,

PATH-COST = *parent*.PATH-COST + *problem*.STEP-COST(*parent*.STATE, *action*)

Implementation: Frontier and Explored

Frontier/Fringe

- Implemented as a Queue:
 - First-in-First-Out, FIFO (aka “queue”): $O(1)$ access
 - Last-in-First-Out, LIFO (aka “stack”): $O(1)$ access
 - Best-First-out (aka “priority queue”): $O(\log(n))$ access
- Two primitives:
 - `ISEMPTY(Queue)`: returns true iff there are no more elements
 - `POP(Queue)`: removes and returns the first element of the queue
 - `INSERT(ELEMENT, Queue)`: inserts an element into queue

Explored

- Implemented as a Hash Table: $O(1)$
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- Choice of hash function critical for efficiency

Implementation: Frontier and Explored

Frontier/Fringe

- Implemented as a Queue:
 - First-in-First-Out, FIFO (aka “queue”): $O(1)$ access
 - Last-in-First-Out, LIFO (aka “stack”): $O(1)$ access
 - Best-First-out (aka “priority queue”): $O(\log(n))$ access
- Two primitives:
 - `ISEMPTY(Queue)`: returns true iff there are no more elements
 - `POP(Queue)`: removes and returns the first element of the queue
 - `INSERT(ELEMENT, Queue)`: inserts an element into queue

Explored

- Implemented as a Hash Table: $O(1)$
- access Three primitives:
 - `ISTHERE(ELEMENT, HASH)`: returns true iff element is in the hash
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Uninformed vs. Informed Search Strategies

Strategies: Two possibilities

- Uninformed strategies (a.k.a. blind strategies)
 - do not use any domain knowledge
 - apply rules arbitrarily and do an exhaustive search strategy

⇒ impractical for some complex problems.
- Informed strategies
 - use domain knowledge
 - apply rules following heuristics (driven by domain knowledge)

⇒ practical for many complex problems.

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Evaluating Search Strategies

- Strategies are evaluated along the following dimensions:

- completeness**: does it always find a solution if one exists?

- time complexity**: how many steps to find a solution?

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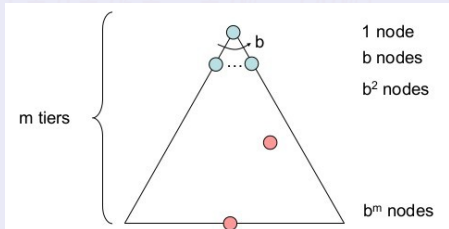
- Time and space complexity are measured in terms of

- b : maximum branching factor of the search tree

- d : depth of the least-cost solution

- m : maximum depth of the state space (may be $+\infty$)

\Rightarrow # nodes: $1 + b + b^2 + \dots + b^m = O(b^{m+1})$



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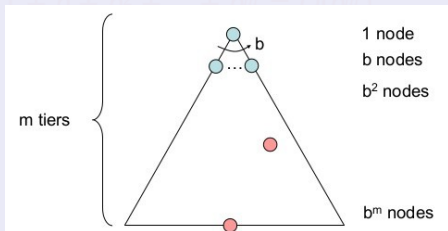
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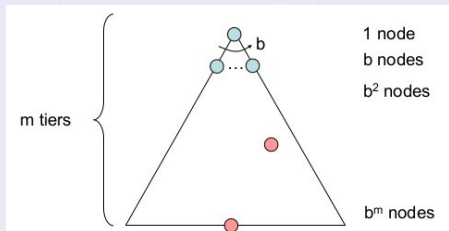
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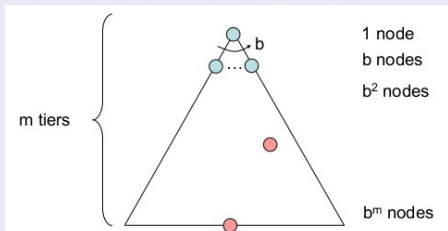
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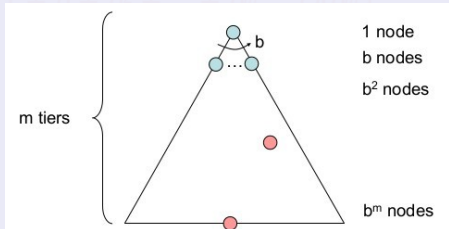
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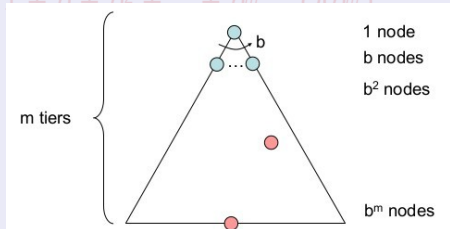
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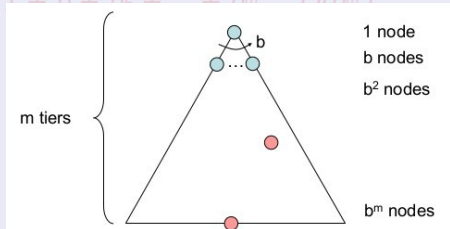
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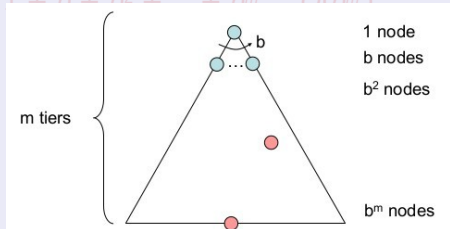


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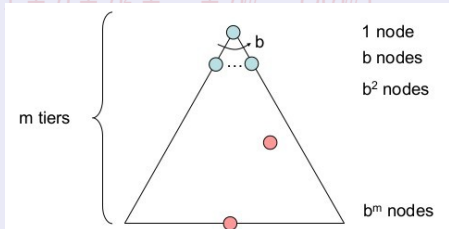
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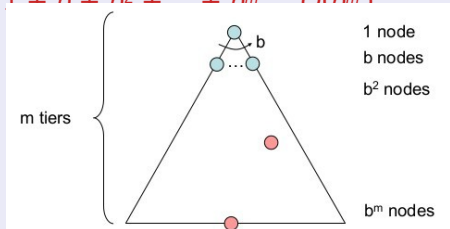
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Uninformed Search Strategies

Uninformed strategies

Use only the information available in the problem

- definition Different uninformed search strategies

- Breadth-first search

- Uniform-cost search

- Depth-first search

- Depth-limited search &

- Iterative-deepening search

Defined by the access strategy of the frontier/fringe
(i.e. the order of node expansion)

Outline

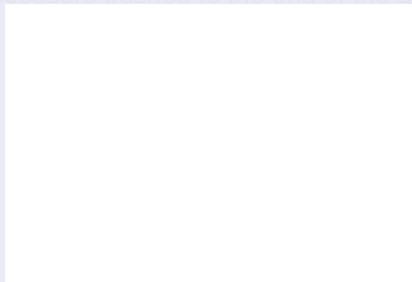
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Breadth-First Search

- Idea: Expand first the shallowest unexpanded nodes
- Implementation: frontier/fringe implemented as a FIFO queue

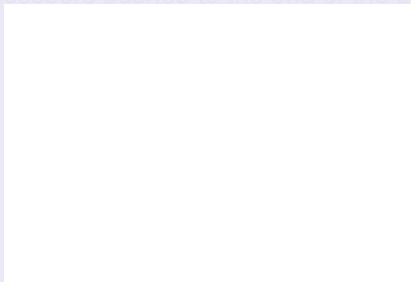
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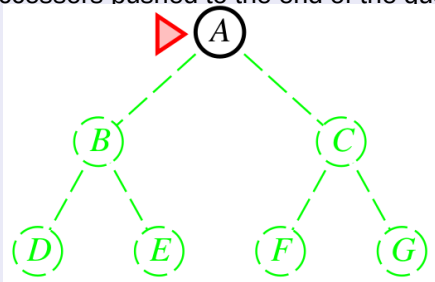
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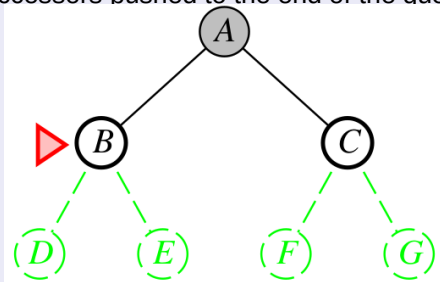


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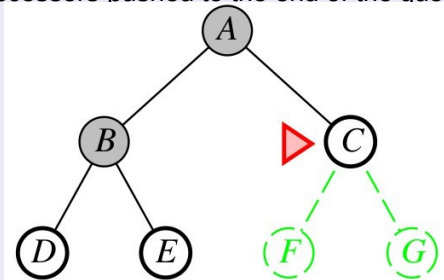


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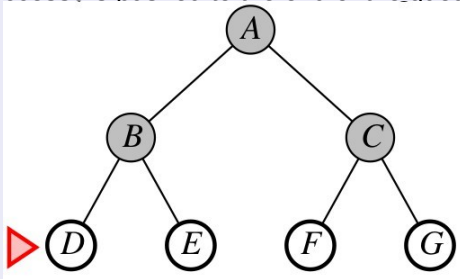


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Breadth-First Search Strategy (BFS) [cont.]

BFS, Graph version (Tree version without

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node  $\leftarrow$  a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier  $\leftarrow$  a FIFO queue with node as the only element
  explored  $\leftarrow$  an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node  $\leftarrow$  POP(frontier) /* chooses the shallowest node in frontier */
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child  $\leftarrow$  CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
        frontier  $\leftarrow$  INSERT(child, frontier)
```

(Oc S. Russell & P. Norwig, AIMA)

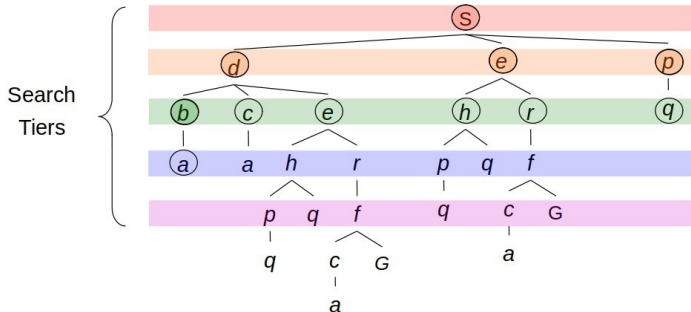
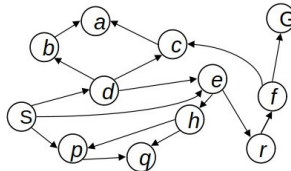
Note: the goal test is applied to each node **when it is generated**, rather than when **it is selected for expansion**
 \Rightarrow solution detected 1 layer earlier

Breadth-First Search: Tiers

State space is explored by tiers (tree)

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue



Breadth-First Search (BFS): Properties

d : depth of shallowest

- solution How many steps?

processes all nodes above shallowest solution

- \Rightarrow takes $O(b^d)$ time

How much memory?

max frontier size: b^d nodes

- $\Rightarrow O(b^d)$ memory size

Is it complete?

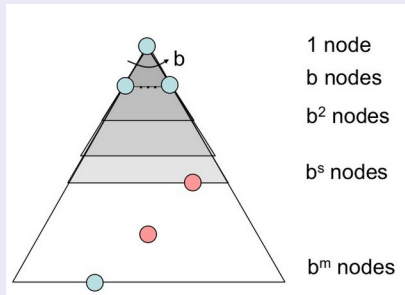
if solution exists, b^d finite

- \Rightarrow Yes

Is it optimal?

if and only if all costs are 1

\Rightarrow shallowest solution



(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

Memory requirement is a major problem for breadth-first search

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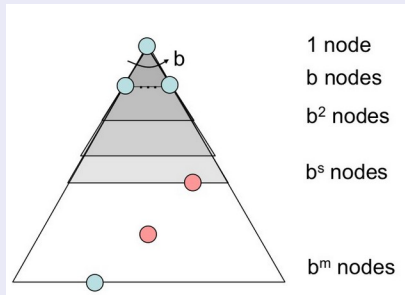
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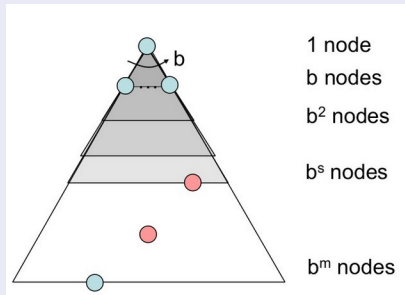
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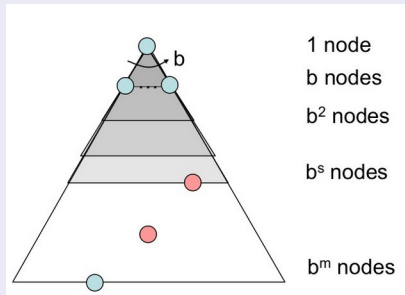
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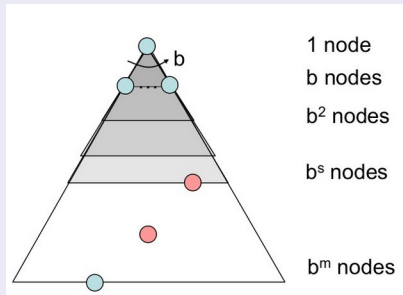
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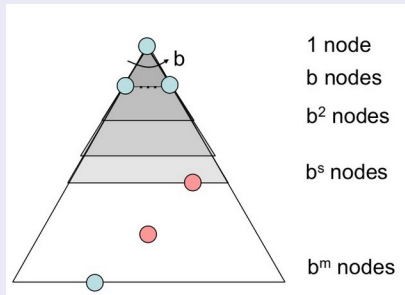
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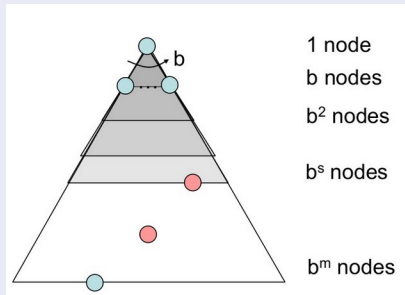
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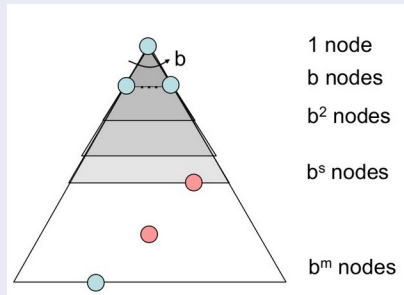
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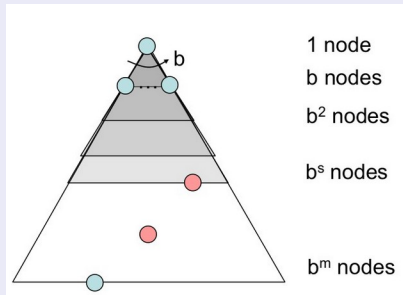
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Breadth-First Search (BFS): Time and Memory

- Assume:

- 1 million nodes generated per second
- 1 node requires 1000 bytes of storage
- branching factor $b = 10$

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

(Oc S. Russell & P. Norwig, AIMA)

Memory requirements bigger problem for BFS than execution time

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Uniform-Cost Search

- Idea: Expand first the node with lowest path cost $g(n)$
- Implementation: frontier/fringe implemented as a priority queue ordered by $g()$
 - ⇒ novel nearest successors pushed to the top of the queue
- similar to BFS if step costs are all equal

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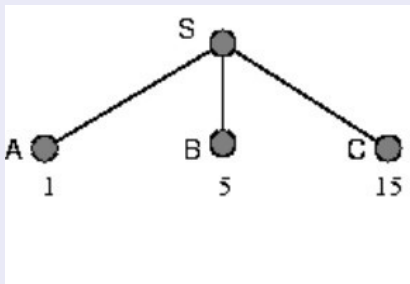
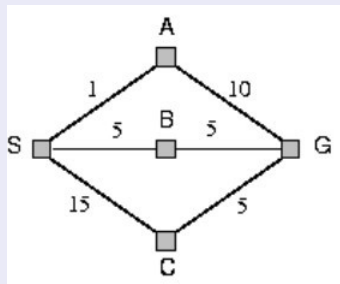
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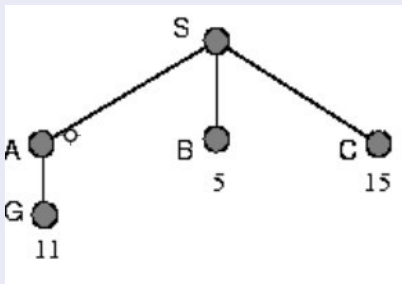
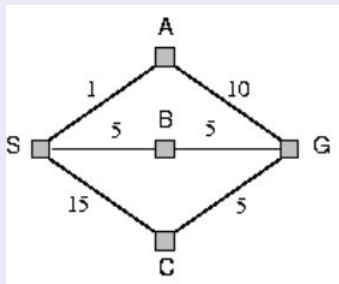


(Courtesy of Michela Milano, UniBO)

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 - \Rightarrow novel nearest successors pushed to the top of the queue
- similar to BFS if step costs are all equal

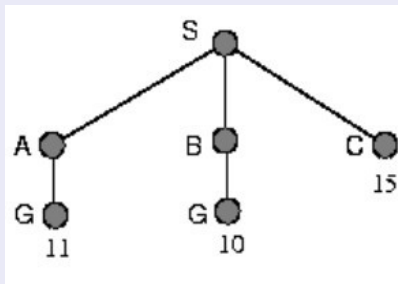
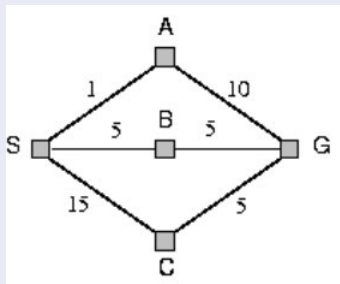


(Courtesy of Michela Milano, UniBO)

Uniform-Cost Search Strategy (UCS)

Uniform-Cost Search

- Idea: Expand first the node with lowest path cost $g(n)$
- Implementation: frontier/fringe implemented as a priority queue ordered by $g()$
 - \Rightarrow novel nearest successors pushed to the top of the queue
- similar to BFS if step costs are all equal



(Courtesy of Michela Milano, UniBO)

Uniform-Cost Search Strategy (UCS) [cont.]

UCS, Graph version (Tree version without

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node  $\leftarrow$  a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier  $\leftarrow$  a priority queue ordered by PATH-COST, with node as the only element
  explored  $\leftarrow$  an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node  $\leftarrow$  POP(frontier) /* chooses the lowest-cost node in frontier */
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child  $\leftarrow$  CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        frontier  $\leftarrow$  INSERT(child, frontier)
      else if child.STATE is in frontier with higher PATH-COST then
        replace that frontier node with child
```

(Oc S. Russell & P. Norwig, AIMA)

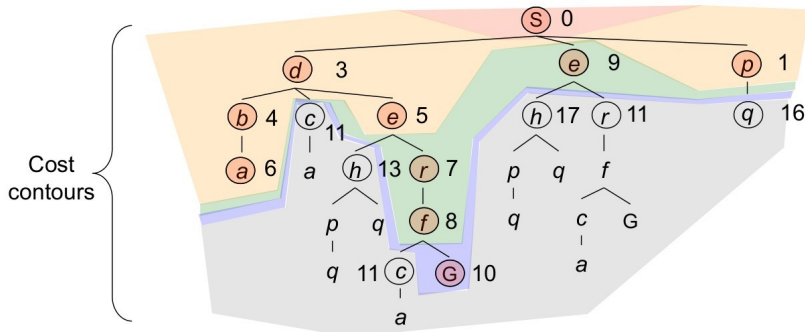
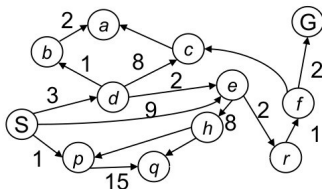
- apply the goal test to a node **when it is selected for expansion** rather than **when it is first generated**
- replace in the frontier a node with same state but worse path cost

• avoid generating suboptimal paths

Uniform-Cost Search

Strategy: expand a
cheapest node first:

Fringe is a priority queue
(priority: cumulative cost)



Uniform-Cost Search (UCS): Properties

C^* : cost of cheapest solution; c : minimum arc cost

$\Rightarrow 1 + \lceil C^*/c \rceil$ “effective depth” How many steps?

processes all nodes costing less than cheapest solution

\Rightarrow takes $O(b^{1+C^*/c})$ time

How much memory?

max frontier size: $b^{1+C^*/c}$

$\Rightarrow O(b^{1+C^*/c})$ memory size

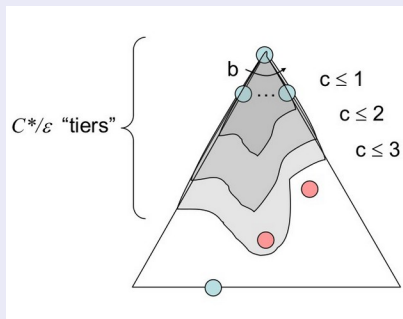
Is it complete?

if solution exists, finite cost

\Rightarrow Yes

Is it optimal?

Yes



(Coc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

Memory requirement is a major problem also for uniform-cost search

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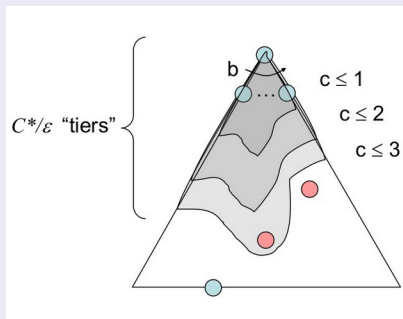
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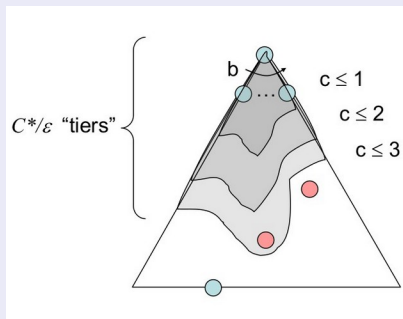
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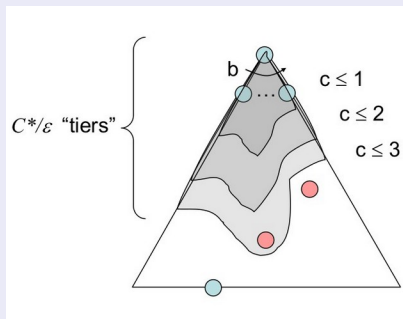
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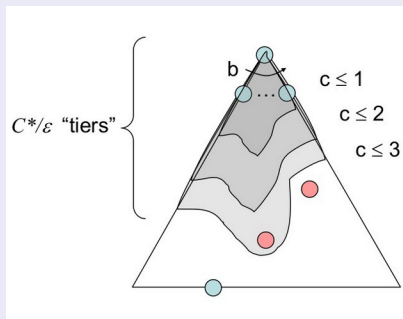
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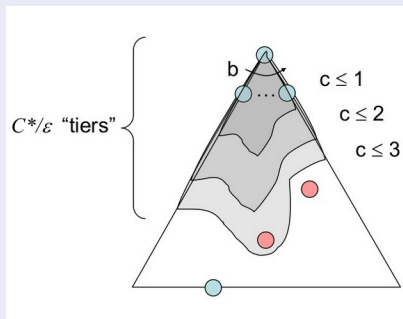
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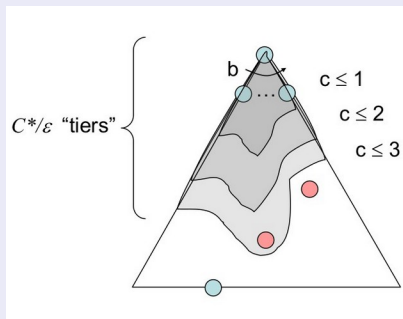
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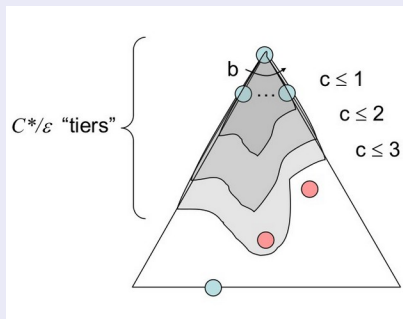
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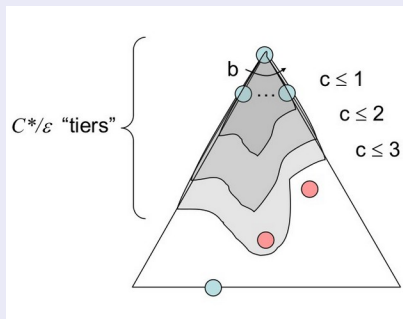
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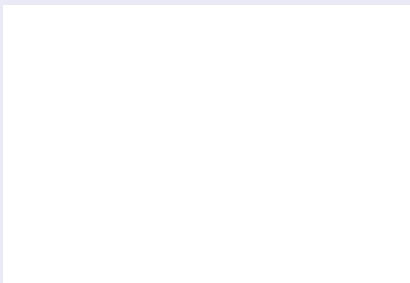
Outline

- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
- 4 Uninformed Search Strategies
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 Informed Search Strategies
 - Greedy Search
 - A* Search Heuristic Functions

Depth-First Search Strategy (DFS)

Depth-First Search

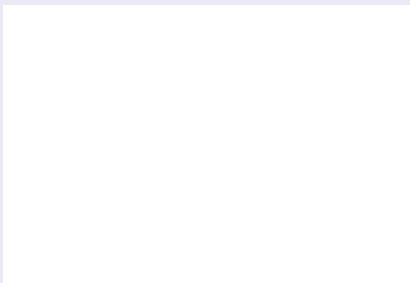
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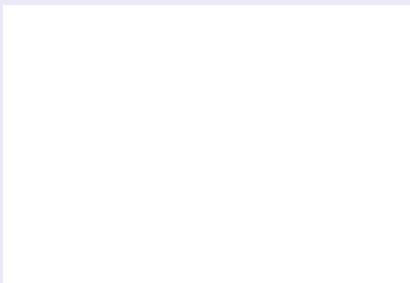
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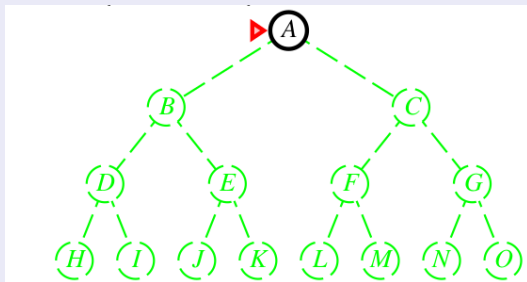
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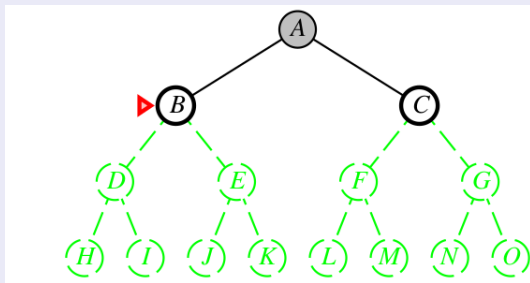
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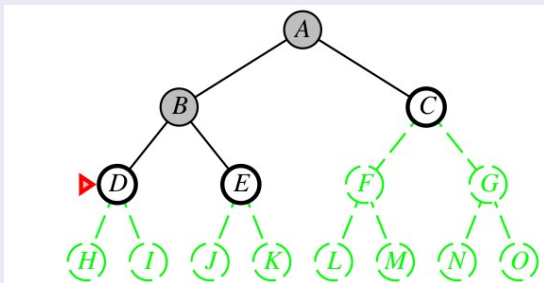
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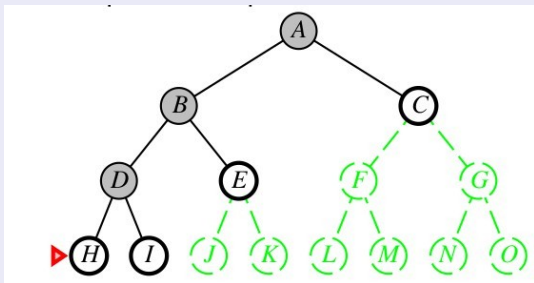
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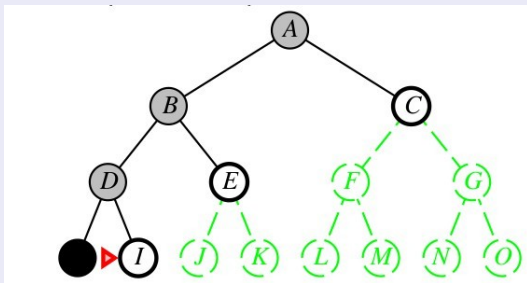
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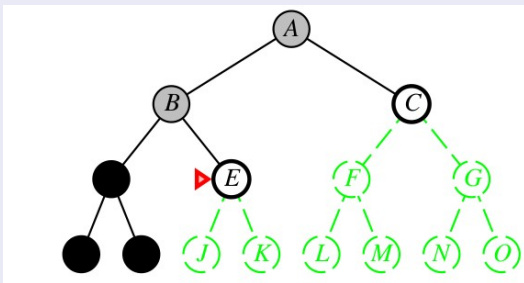
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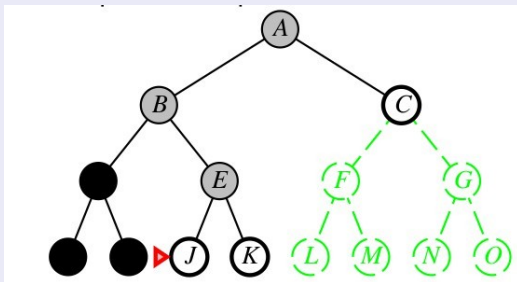
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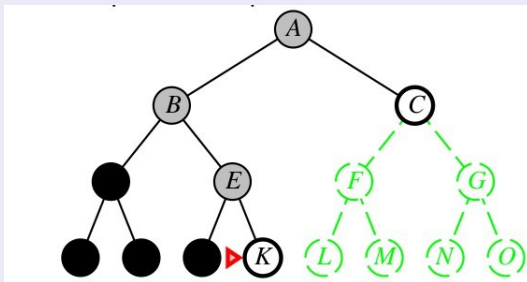
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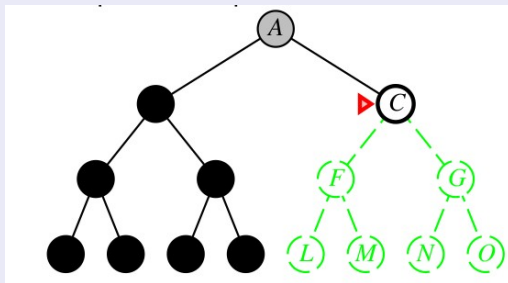
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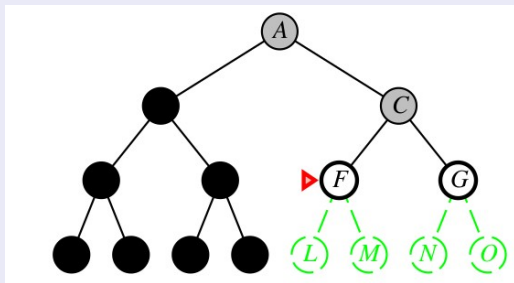
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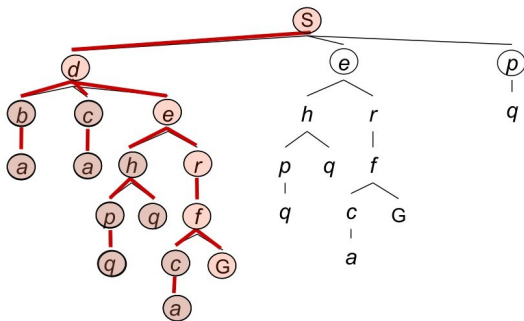
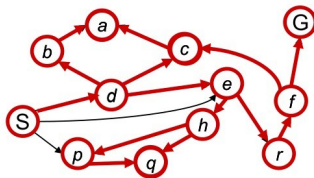
DFS on a Graph

Similar to BFS, using a LIFO access for frontier/fringe rather than FIFO.

Depth-First Search

Strategy: expand a deepest node first

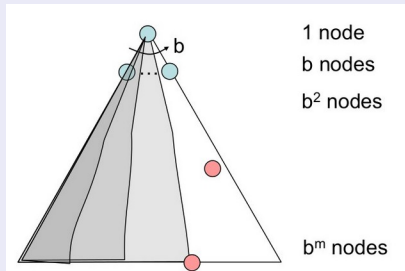
Implementation: Fringe is a LIFO stack



Depth-First Search (DFS): Properties

- How many steps?
 - could process the whole tree! \Rightarrow if m finite, takes $O(b^m)$ time

Is it complete?

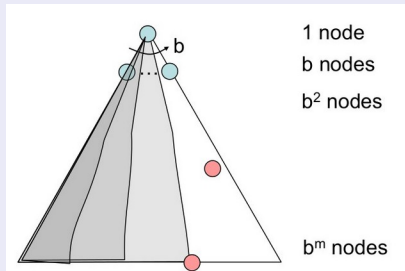


(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

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 - only siblings on path to root
 - $\Rightarrow O(bm)$ memory size

Is it complete?



(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

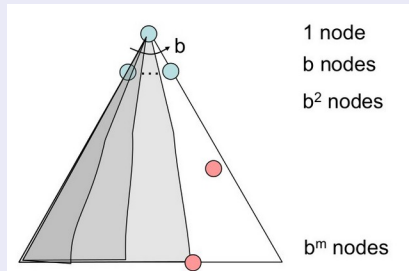
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Is it complete?

- if infinite state space: no
- if finite state space:

- Is it optimal?
 - No, regardless of depth/cost
 - \Rightarrow "leftmost" solution



(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

Memory requirement much better than BFS: $O(bm)$ vs. $O(b^d)$!

Depth-First Search (DFS): Properties

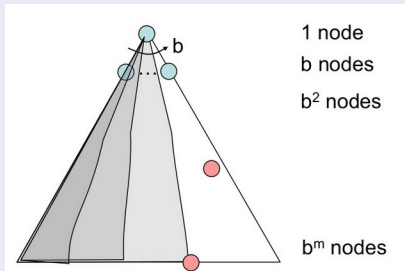
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Is it complete?

- if infinite state space: no
- if finite state space:
 - graph version: yes
 - tree version: only if we prevent loops

Is it optimal?

No, regardless of



(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

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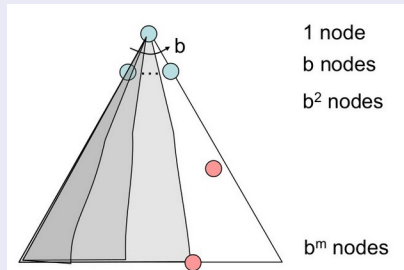
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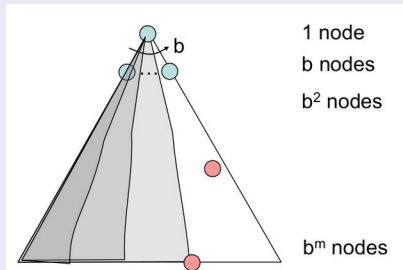
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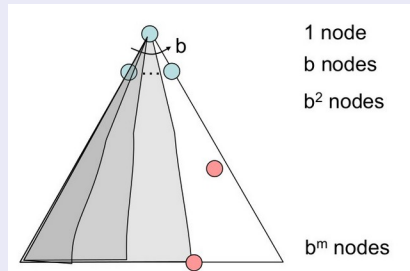
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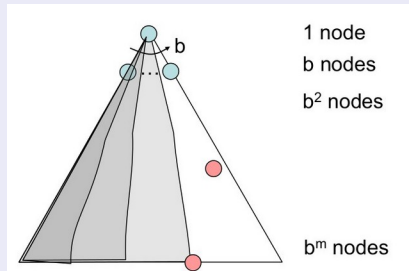
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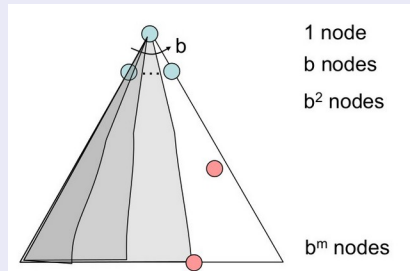
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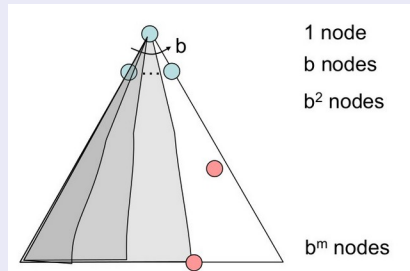
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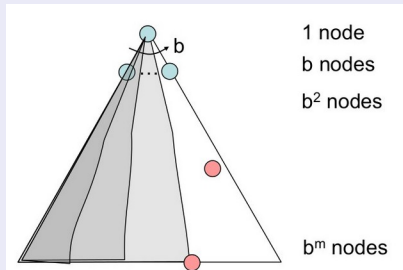
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- if infinite state space: no
- if finite state space:
 - graph version: yes
 - tree version: only if we prevent loops

Is it optimal?

No, regardless of



(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

Memory requirement much better than BFS: $O(bm)$ vs. $O(b^d)$!
 \Rightarrow typically preferred to BFS

A Variant of DFS: Backtracking Search

Backtracking Search

- Idea: **only one successor is generated at the time**
 - each partially-expanded node remembers which successor to generate next
 - generate a successor by modifying the current state description, rather than copying it first
 - Applied in CSP, SAT/SMT and Logic Programming

\Rightarrow only $O(m)$ memory is needed rather than $O(bm)$

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- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
- 4 Uninformed Search Strategies
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 Informed Search Strategies
 - Greedy Search
 - A* Search Heuristic Functions

Depth-Limited Search (DLS) Strategy

Depth-Limited Search (DLS)

- Idea: **depth-first search with depth limit l**
 - i.e., nodes at depth l treated as having no successors
 - DFS is DLS with $l = +\infty$
- solves the infinite-path problem of DFS**
 \Rightarrow allows DFS deal with infinite-state spaces
- useful also if maximum-depth is known by domain knowledge**
 - e.g., if maximum node distance in a graph (**diameter**) is known
 - Ex: **Romania trip**: 9 steps

Drawbacks (d : depth of the shallowest goal):

if $d > l \Rightarrow$ incomplete

if $d < l \Rightarrow$ takes $O(b^l)$ instead of $O(b^d)$ steps

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Depth-Limited Search (DLS) Strategy [cont.]

Recursive DLS

function DEPTH-LIMITED-SEARCH(*problem*, *limit*) **returns** a solution, or failure/cutoff
return RECURSIVE-DLS(MAKE-NODE(*problem*.INITIAL-STATE), *problem*, *limit*)

function RECURSIVE-DLS(*node*, *problem*, *limit*) **returns** a solution, or failure/cutoff
if *problem*.GOAL-TEST(*node*.STATE) **then return** SOLUTION(*node*)

else if *limit* = 0 **then return** *cutoff*

else

cutoff_occurred? \leftarrow false

for each *action* **in** *problem*.ACTIONS(*node*.STATE) **do**

child \leftarrow CHILD-NODE(*problem*, *node*, *action*)

result \leftarrow RECURSIVE-DLS(*child*, *problem*, *limit* - 1)

if *result* = *cutoff* **then** *cutoff_occurred?* \leftarrow true

else if *result* \neq *failure* **then return** *result*

if *cutoff_occurred?* **then return** *cutoff* **else return** *failure*

(Oc S. Russell & P. Norwig, AIMA)

Iterative-Deepening Search Strategy (IDS)

Iterative-Deepening Search

- Idea: call iteratively DLS for increasing depths $l = 0, 1, 2, 3, \dots$
- combines the advantages of breadth- and depth-first strategies
 - complete (like BFS)
 - takes $O(b^d)$ steps (like BFS and DFS)
 - requires $O(bd)$ memory (like DFS)
 - explores a single branch at a time (like DFS)
 - optimal only if step cost = 1
- optimal variants exist: iterative-lengthening search (see AIMA)

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
  for depth = 0 to  $\infty$  do
    result  $\leftarrow$  DEPTH-LIMITED-SEARCH(problem, depth)
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Iterative-Deepening Search: Example

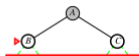
Limit = 0



(Oc S. Russell & P. Norwig, AIMA)

Iterative-Deepening Search: Example

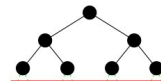
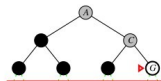
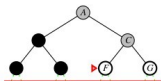
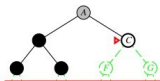
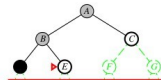
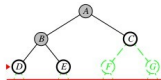
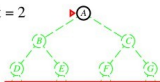
Limit = 1



(Oc S. Russell & P. Norwig, AIMA)

Iterative-Deepening Search: Example

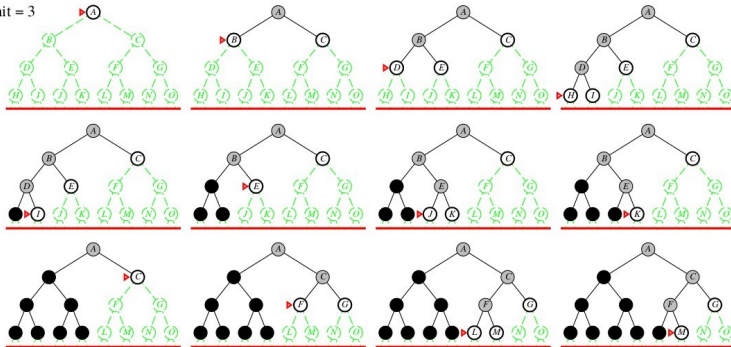
Limit = 2



(Oc S. Russell & P Norwig, AIMA)

Iterative-Deepening Search: Example

Limit = 3



(Oc S. Russell & P. Norwig, AIMA)

Iterative-Deepening Search Strategy (IDS) [cont.]

Remark: Why “only” $O(b^d)$ steps?

- may seem wasteful since states are generated multiple times...
- ... however, only a small fraction of nodes are multiply generated
- number of repeatedly-generated nodes decreases exponentially with number of repetitions
 - depth 1 (b nodes): repeated d times
 - depth 2 (b^2 nodes): repeated $d - 1$ times
 - ...
 - depth d (b^d nodes): repeated 1 time

\Rightarrow The total number of generated nodes is:

$$\begin{aligned} N(\text{IDS}) &= (d)b^1 + (d-1)b^2 \\ &+ \dots + (1)b^d = O(b^d) \end{aligned}$$

$$\begin{aligned} \bullet \quad N(\text{BFS}) &= b^1 + b^2 + \dots + b^d \\ &= O(b^d) \end{aligned}$$

Ex: with $b = 10$ and $d = 5$:

$$\begin{aligned} N(\text{IDS}) &= 50 + 400 + 3,000 + 20,000 + 100,000 = 123,000 \end{aligned}$$

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Bidirectional Search [hints]

- Idea: **Two simultaneous searches:**

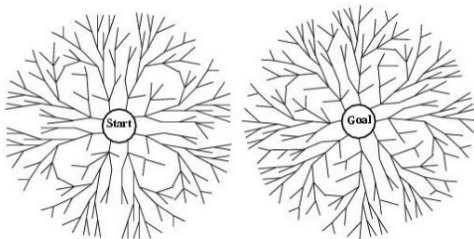
- **forward:** from start node
- **backward:** from goal node

checking if the node belongs to the
other frontier before expansion

Rationale: $b^{d/2} + b^{d/2} \ll b^d$

- \Rightarrow number of steps and memory consumption are $\approx 2b^{d/2}$

backward
some cas



Uninformed Search Strategies: Comparison

Evaluation of tree-search strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes ^a	Yes ^{a,b}	No	No	Yes ^a	Yes ^{a,d}
Time	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes ^c	Yes	No	No	Yes ^c	Yes ^{c,d}

^a: complete if b is finite

^b: complete if step costs $\geq E$ for some positive E

^c: optimal if step costs are all identical

^d: if both directions use breadth-first search

(Oc S. Russell & P. Norwig, AIMA)

For graph searches, the main differences are:

- depth-first search is complete for finite-state spaces
- space & time complexities are bounded by the state space size

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Informed Search Strategies

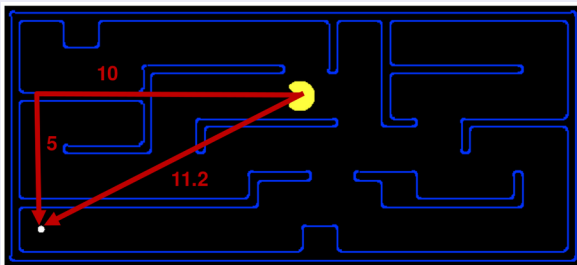
Some general principles

- The intelligence of a system cannot be measured only in terms of search capacity, but in **the ability to use knowledge about the problem to reduce/mitigate the combinatorial explosion**
- If the system has some control on the order in which candidate solutions are generated, then it is useful to use this order so that actual solutions have a high chance to appear earlier
- **Intelligence, for a system with limited processing capacity, is the wise choice of what to do next**

Heuristic search and heuristic functions

Heuristic search and heuristic functions

- Idea: **don't ignore the goal when selecting nodes**
- Intuition: often there is **extra knowledge** that can be used to **guide the search towards the goal**: **heuristics**
- A **heuristic** is
 - a function $h(n)$ that estimates how close a state n is to a goal
 - designed for a particular search problem
 - Ex **Manhattan distance**, **Euclidean distance** for pathing



Best-first Search Strategies

General approach of informed search: Best-first search

- **Best-first search**: node selected for expansion based on an **evaluation function $f(n)$**
 - represent a cost estimate \Rightarrow choose node which appears best
 - implemented like uniform-cost search, with f instead of g \Rightarrow the frontier is a priority queue sorted in decreasing order of $h(n)$
 - both tree-based and graph-based versions
 - most often f includes a **heuristic function $h(n)$**
- **Heuristic function $h(n) \in \mathbb{R}^+$: estimated cost of the cheapest path from the state at node n to a goal state**
 - $h(n) \geq 0 \forall n$
 - If G is goal, then $h(G) = 0$
 - implements **extra domain knowledge**
 - **depends only on state, not on node**
(e.g., independent on paths)

Main strategies:

- Greedy search
- A* search

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(e.g., independent on paths)

Main strategies:

- Greedy search
- A* search

Best-first Search Strategies

General approach of informed search: Best-first search

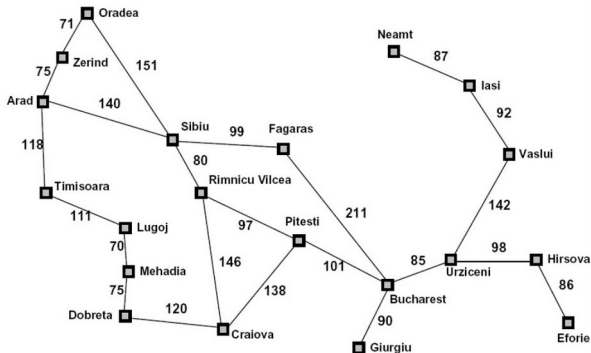
- **Best-first search**: node selected for expansion based on an **evaluation function $f(n)$**
 - represent a cost estimate \Rightarrow choose node which appears best
 - implemented like uniform-cost search, with f instead of g \Rightarrow the frontier is a priority queue sorted in decreasing order of $h(n)$
 - both tree-based and graph-based versions
 - most often f includes a **heuristic function $h(n)$**
- **Heuristic function $h(n) \in \mathbb{R}^+$: estimated cost of the cheapest path from the state at node n to a goal state**
 - $h(n) \geq 0 \forall n$
 - If G is goal, then $h(G) = 0$
 - implements **extra domain knowledge**
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Main strategies:

- **Greedy search**
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Example: Straight-Line Distance $h_{SLD}(n)$

- $h(n) \stackrel{\text{def}}{=} h_{SLD}(n)$: straight-line distance heuristic
 - different from actual minimum-path distance
 - cannot be computed from the problem description itself



Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
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$h(x)$

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Greedy Best-First Search Strategy (GBFS)

Greedy Best-First Search (aka Greedy Search)

- Idea: Expand first the node n with lowest estimate cost to the closest goal, $h(n)$
 - Implementation: same as uniform-cost search, with $g(n) \stackrel{\text{def}}{=} h(n)$
 - \Rightarrow expands the node that appears to be closest to goal
- both tree and graph versions

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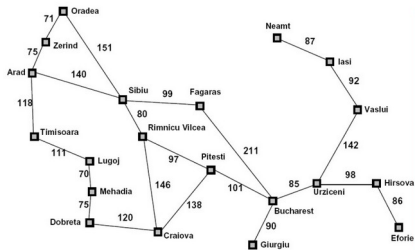
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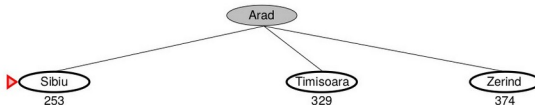
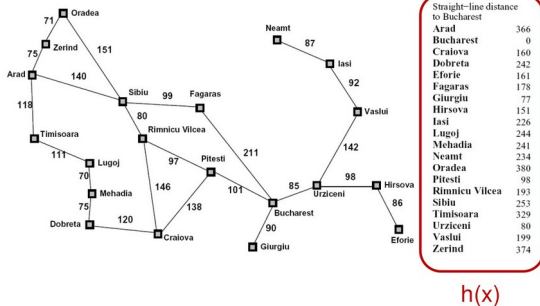
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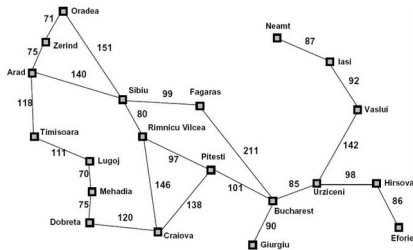
$h(x)$

▶ Arad
366

Greedy Best-First Search Strategy: Example



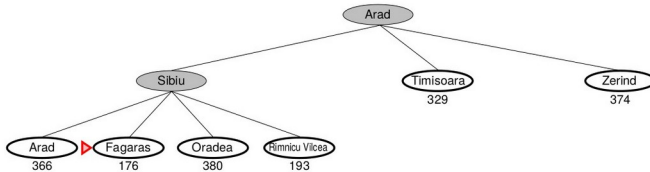
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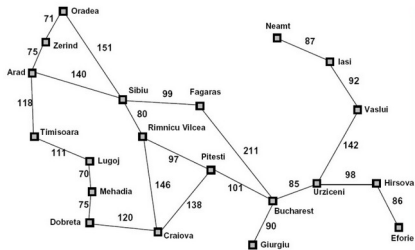
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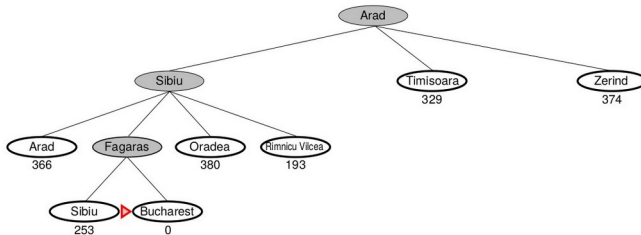
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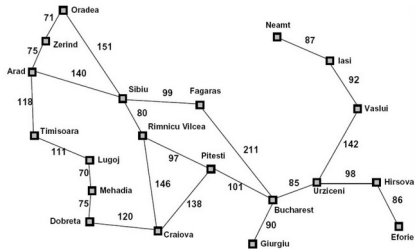
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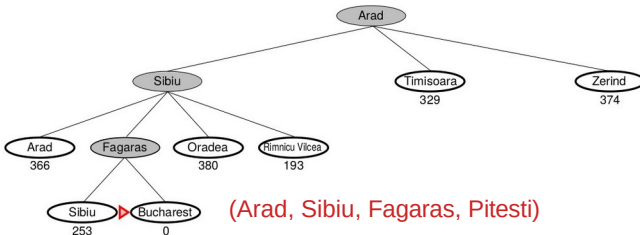
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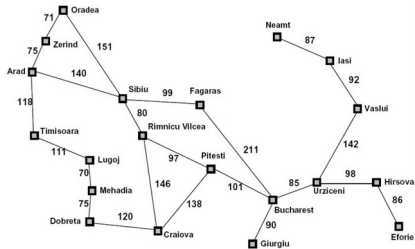
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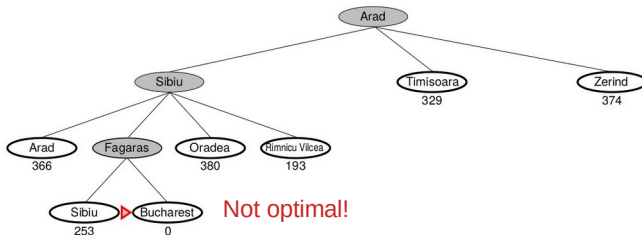
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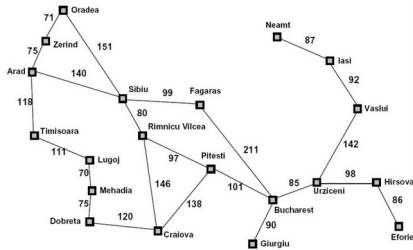
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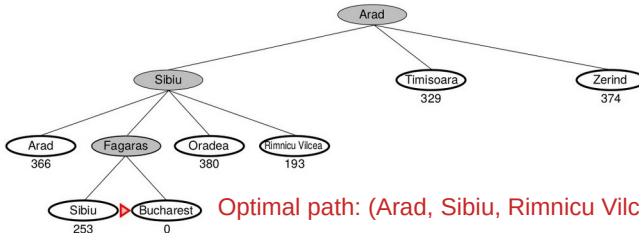
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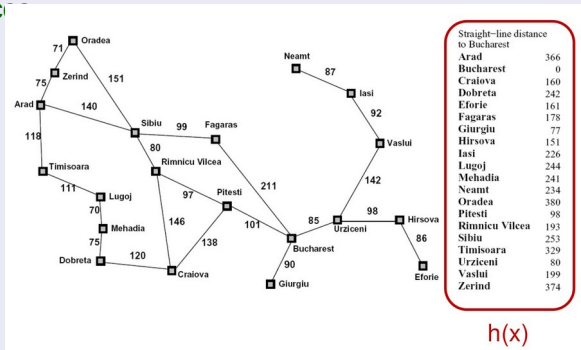
$h(x)$



Optimal path: (Arad, Sibiu, Rimnicu Vilcea, Pitesti)

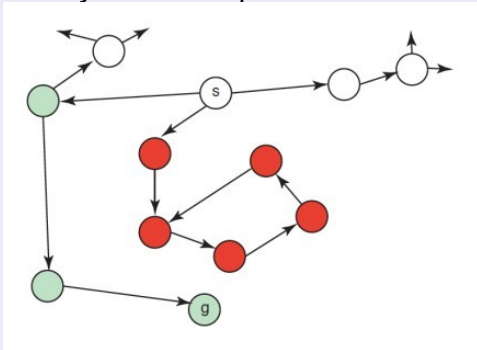
Greedy Best-First Search: (Non-)Optimality

- Greedy best-first search is not optimal
 - it is not guaranteed to find the best solution
 - it is not guaranteed to find the best path toward a solution
- picks the node with minimum (estimated) distance to goal, regardless the cost to reach it
 - Ex: when in Sibiu, it picks Fagaras rather than Rimnicu Vilcea



Greedy Best-First Search: (In-)Completeness

- Tree-based Greedy best-first search is not complete
- may lead to infinite loops
- Graph-based version complete (if state space finite) substantially same completeness issues as DFS



Greedy Best-First Search (GBFS): Properties

- How many steps?
 - in worst cases may explore all states
 \Rightarrow takes $O(b^d)$ time
 - if good heuristics:
 \Rightarrow may give good improvements

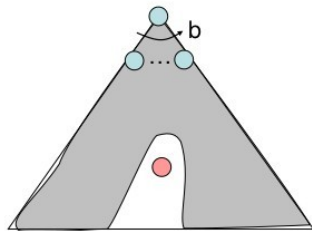
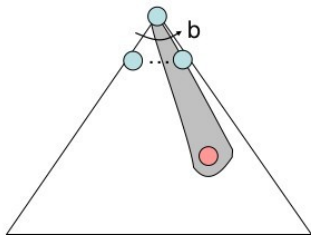
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 - max frontier size: b^d
 $\Rightarrow O(b^d)$ memory size

Is it complete?

tree: no

graph: yes if space finite

Is it optimal?



(Oc. D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

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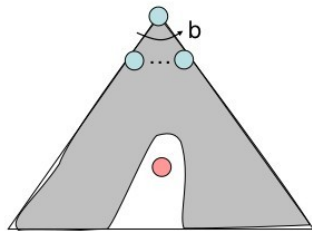
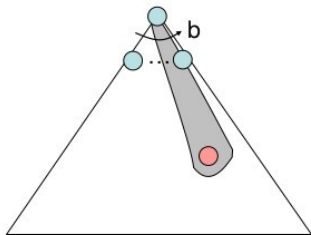
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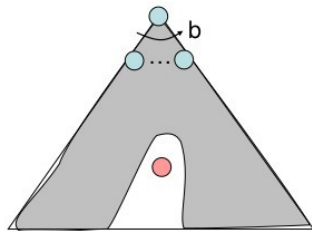
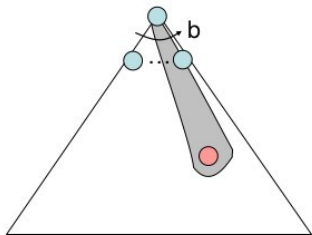
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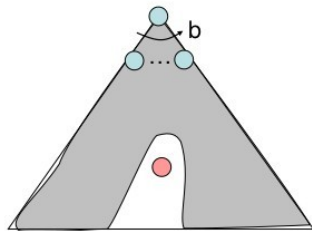
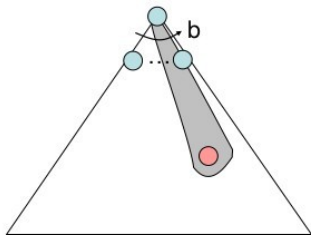
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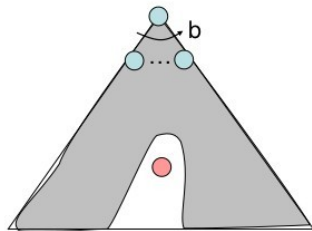
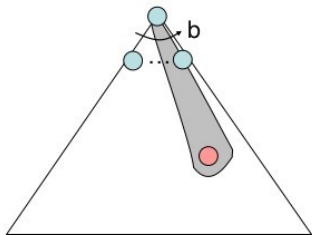


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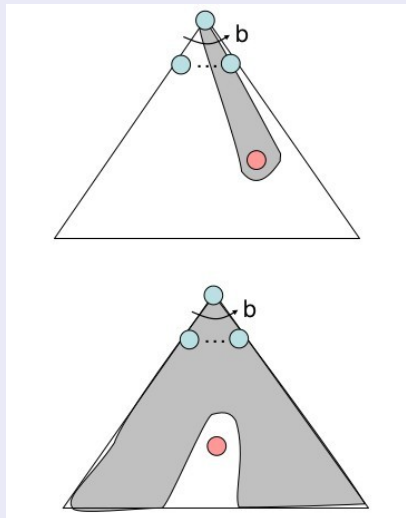
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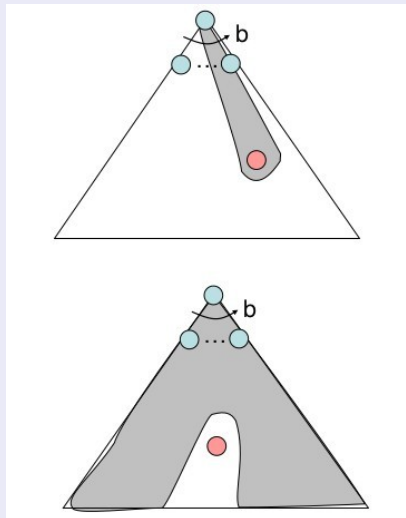
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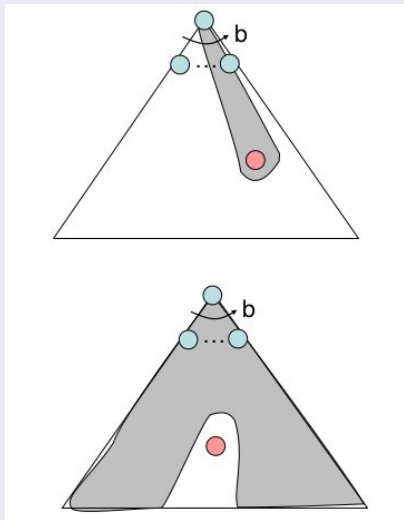
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A* Search Strategy

A* Search

- Best-known form of best-first search
- Idea: avoid expanding paths that are already expensive
- Combine Uniform-Cost and Greedy search: $f(n) = g(n) + h(n)$
 - $g(n)$: cost so far to reach n
 - $h(n)$: estimated cost to goal from n
 - $f(n)$: estimated total cost of path through n to goal

⇒ Expand first the node n with lowest estimated cost of

- the cheapest solution through n

Implementation: same as uniform-cost search, with $g(n) + h(n)$

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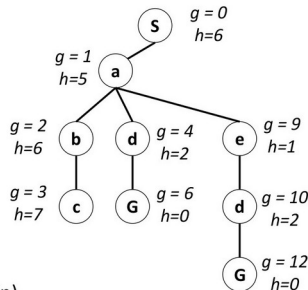
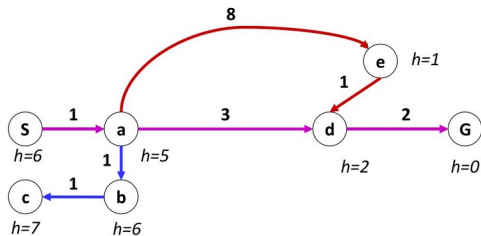
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A* Search Strategy: Example

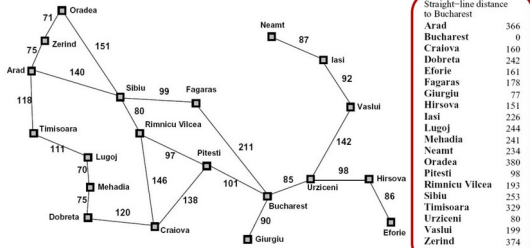
- Uniform-cost orders by path cost, or *backward cost* $g(n)$
- Greedy orders by goal proximity, or *forward cost* $h(n)$



- A* Search orders by the sum: $f(n) = g(n) + h(n)$

(Oc: D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

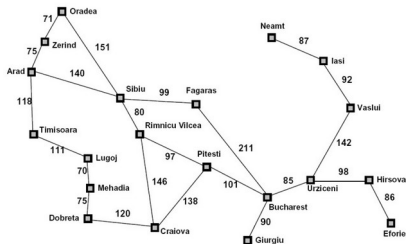
A* Search Strategy: Example



$h(x)$

▶ Arad
366=0+366

A* Search Strategy: Example

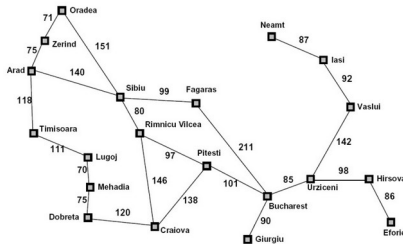


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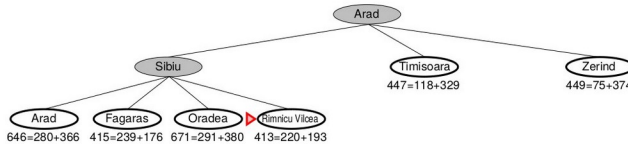


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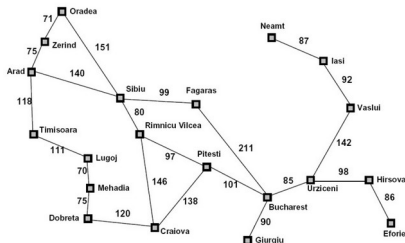


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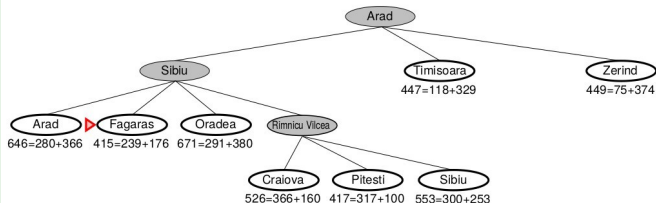
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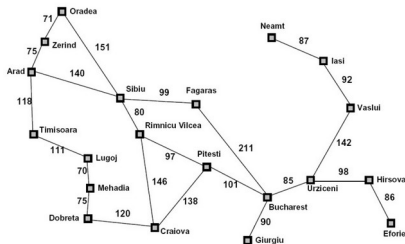
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Vaslui	199
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$h(x)$

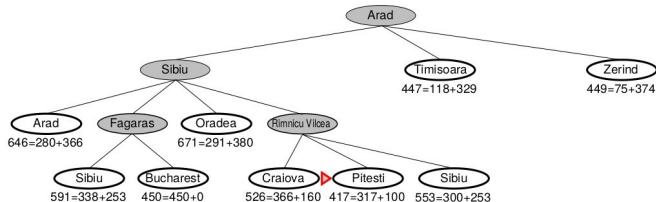


A* Search Strategy: Example

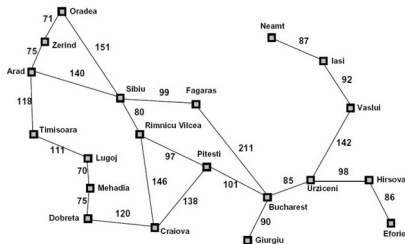


Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
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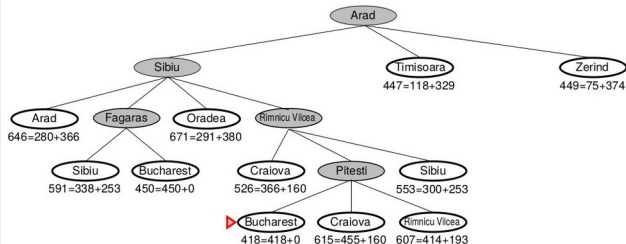


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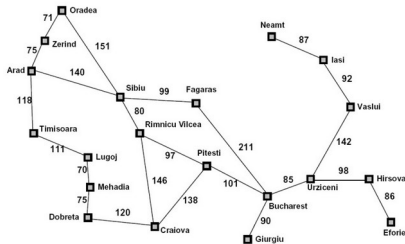


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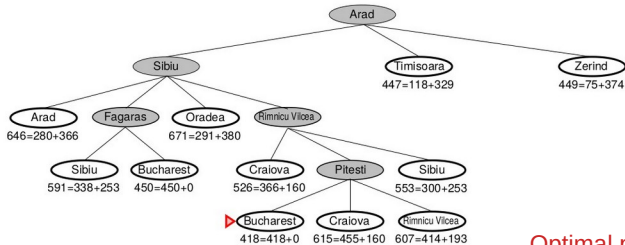
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$h(x)$



Optimal path!

A* Search: Admissible and Consistent Heuristics

Admissible heuristics $h(n)$

- $h(n)$ is **admissible** (aka **optimistic**) iff it never overestimates the cost to reach the goal:
 - $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n
 - ex: the straight-line distance $h_{SDL}()$ to Bucharest

Consistent heuristics $h(n)$

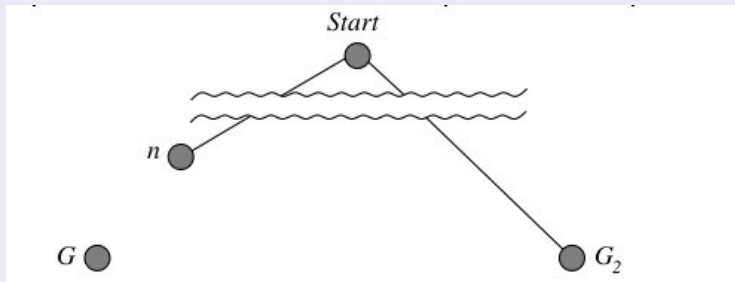
- $h(n)$ is **consistent** (aka **monotonic**) iff, for every successor n' of n generated by any action a with step cost $c(n, a, n^m)$,
 $h(n) \leq c(n, a, n^m) + h(n^m)$
 - verifies the triangular inequality
 - ex: the straight-line distance $h_{SDL}()$ to Bucharest

A* Tree Search: Optimality

If $h(n)$ is admissible, then A* tree search is optimal

- Suppose some sub-optimal goal G_2 is in the frontier queue.
- n unexpanded node on a shortest path to an optimal goal G .
- then:
$$\begin{aligned}f(G_2) &= g(G_2) \quad \text{since } h(G_2) = 0 \\&> g(G) \quad \text{since } G_2 \text{ sub-optimal} \\&\geq f(n) \quad \text{since } h \text{ is admissible}\end{aligned}$$

\Rightarrow A* will not pick G_2 from the frontier queue before n



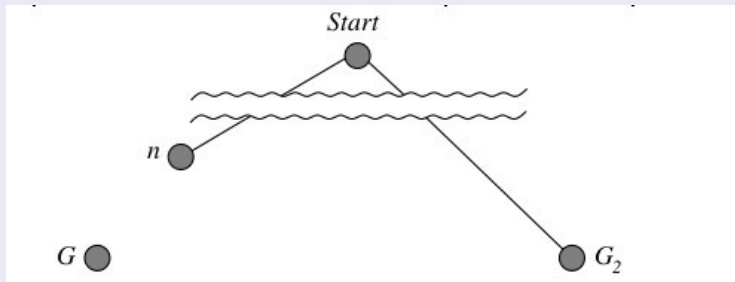
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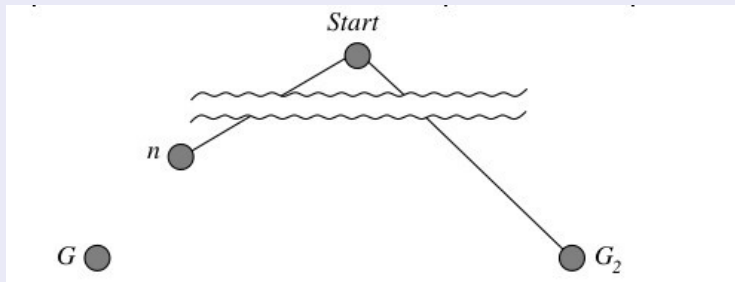


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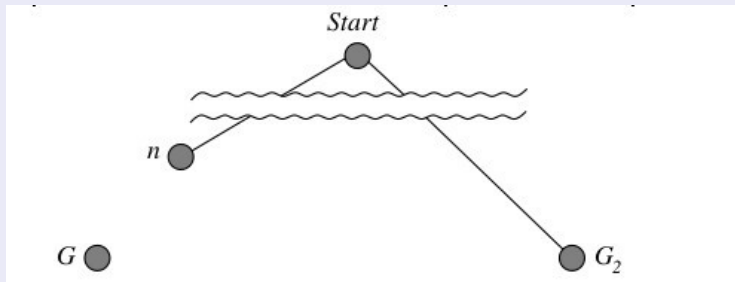


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A* Graph Search: Properties

Properties

- 1 if $h(n)$ is consistent, then $h(n)$ is admissible (straightforward)
- 2 If $h(n)$ is consistent, then $f(n)$ is non-decreasing along any path:
 - $f(n^m) = g(n^m) + h(n^m) = g(n) + c(n, a, n^m) + h(n^m) \geq g(n) + h(n) = f(n)$
- 3

If (Graph) A* selects a node n from the frontier, then the optimal path to that node has been found

if not so, there would be a node n^m in the frontier on the

- optimal path to n (because of the graph separation property)
- since f is non-decreasing along any path, $f(n^m) \leq f(n)$

\Rightarrow since n has been selected before n^m , $f(n^m) < f(n)$



\Rightarrow A* (graph search) expands nodes in non-decreasing order of f

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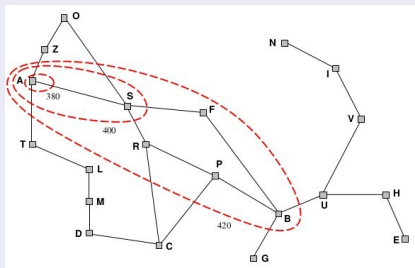


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A* Graph Search: Optimality

If $h(n)$ is consistent, then A* graph search is optimal

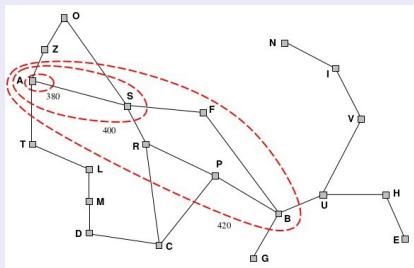
- A* expands nodes in order of non-decreasing f value
- Gradually adds “ f -contours” of nodes (as BFS adds layers)
 - contour i has all nodes with $f = f_i$, s.t. $f_i < f_{i+1}$
 - cannot expand contour f_{i+1} until contour f_i is fully expanded
- If C^* is the cost of the optimal solution path
 - 1 A* expands all nodes s.t. $f(n) < C^*$
 - 2 A* might expand some of the nodes on “goal contour” s.t. $f(n) = C^*$ before selecting a goal node.
 - 3 A* does not expand nodes s.t. $f(n) > C^*$ (pruning)



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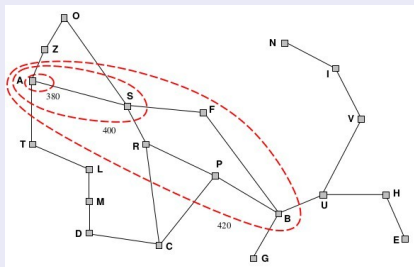
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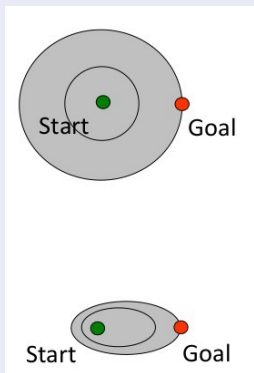
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UCS vs A* Contours

Intuition

- UCS expands equally in all “directions”
- A* expands mainly toward the goal

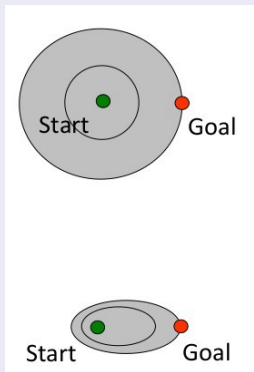


(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

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A* Search: Completeness

If all step costs exceed some finite c and b is finite,
then there are only finitely many nodes n s.t. $f(n) \leq C^*$

\Rightarrow A* is complete.

A* Search: Properties

Let $c =_{\text{def}} (h^+ - h)/h^*$ (relative error)

b^e : effective branching factor

How many steps?

takes $O((b^c)^d)$ time

if good heuristics, may give dramatic improvements

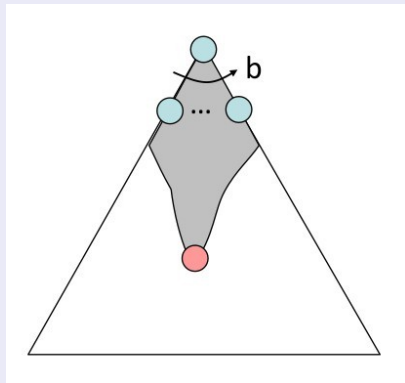
How much memory?

Keeps all nodes in memory
 $\Rightarrow O(b^d)$ memory size (like UCS)

Is it complete?

yes

Is it optimal?



(Oc D. Klein, P. Abbeel, S. Levine, S. Russell, U. Berkeley)

Memory requirement is a major problem also for

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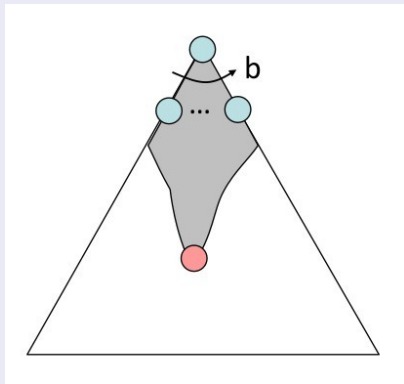
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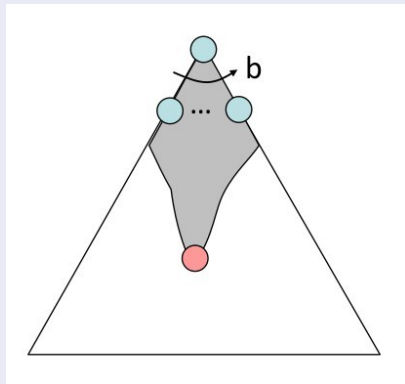
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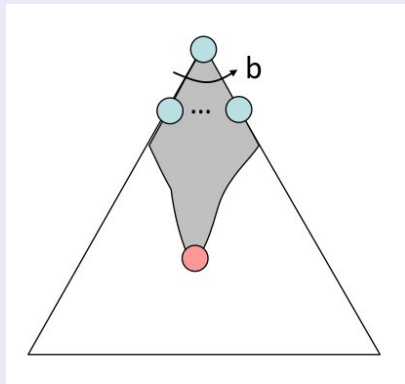
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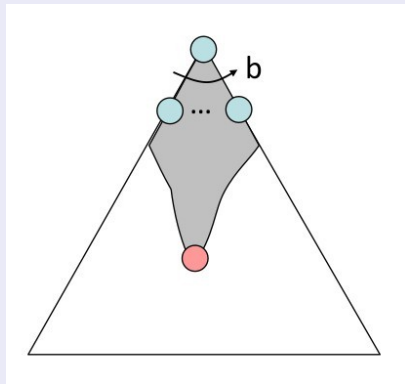
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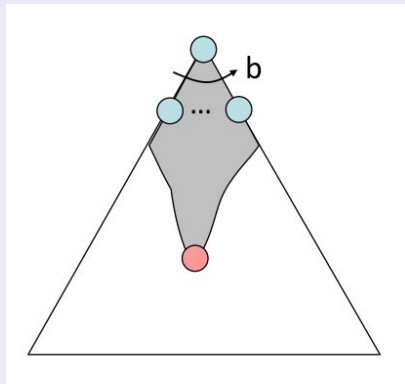
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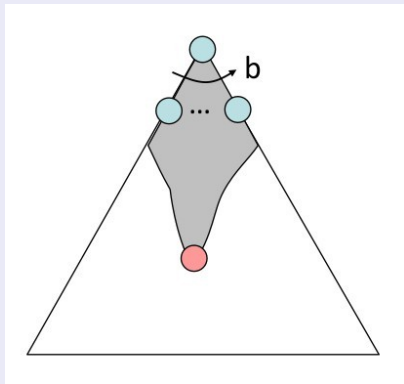
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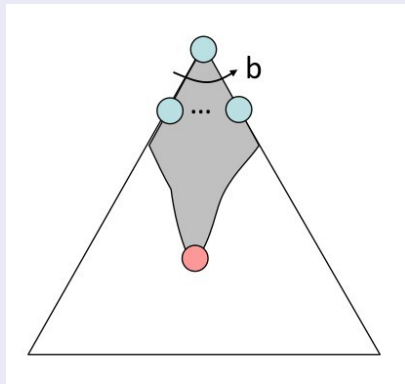
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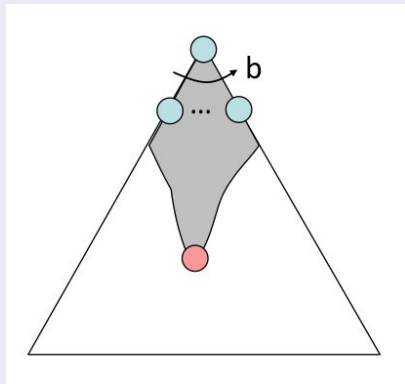
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Memory-bounded Heuristic Search (hints)

Some solutions to A^* space problems (maintain completeness and optimality)

- Iterative-deepening A^* (IDA*)
 - here cutoff information is the f-cost ($g+h$) instead of depth
- Recursive best-first search (RBFS)
 - attempts to mimic standard best-first search with linear space

(simple) Memory-bounded A^* ((S)MA*)

drop the worst-leaf node when memory is full

Outline

- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
- 4 Uninformed Search Strategies
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 Informed Search Strategies
 - Greedy Search
 - A* Search Heuristic Functions

Admissible Heuristics

Main problem

What is the best admissible/consistent heuristic?

Dominance of Admissible Heuristics

Dominance

Let $h_1(n)$, $h_2(n)$ admissible heuristics.

- $h_2(n)$ dominates $h_1(n)$ iff $h_2(n) \geq h_1(n)$ for all n .

$\Rightarrow h_2(n)$ is better for search

- is nearer to $h^*(n)$

Let $h_1(n)$, $h_2(n)$ admissible heuristics. Let $h_{12} \stackrel{\text{def}}{=} \max(h_1(n),$

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h_{12} dominates both $h_1(n)$, $h_2(n)$

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Admissible Heuristics: Example

Ex: Heuristics for the 8-puzzle

- $h_1(n)$: number of misplaced tiles
- $h_2(n)$: total Manhattan distance over all tiles
 - (i.e., # of squares from desired location of each tile)
- $h_1(S)?$ 6
- $h_2(S)?$ $4+0+3+3+1+0+2+1 = 14$
- $h^*(S)?$ 26
- both $h_1(n)$, $h_2(n)$ admissible (\leq number of actual steps to solve)
- $h_2(n)$ dominates $h_1(n)$

7	2	4
5		6
8	3	1

Start State

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4	5	6
7	8	

Goal State

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- $h_2(n)$: total Manhattan distance over all tiles
 - (i.e., # of squares from desired location of each tile)
- $h_1(S)$? 6
- $h_2(S)$? $4+0+3+3+1+0+2+1 = 14$
- $h^*(S)$? 26
- both $h_1(n)$, $h_2(n)$ admissible (\leq number of actual steps to solve)
- $h_2(n)$ dominates $h_1(n)$

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

Admissible Heuristics: Example

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Quality of Heuristics

Effective branching factor

- **Effective branching factor b^*** : the branching factor that a uniform tree of depth d would have in order to contain $N+1$ nodes

$$N + 1 = 1 + b^* + (b^*)^2 + \dots + (b^*)^d$$

- N being the number of nodes generated by the A^*
 - search ex: if $d=5$ and $N = 52$, then $b^* = 1.92$
 - experimental measure of b^* is fairly constant for hard problems
- \Rightarrow can provide a good guide to the heuristic's overall usefulness

Ideal value of b^* is 1

Admissible Heuristics: Example [cont.]

Average performances on 100 random samples of 8-puzzle

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	—	539	113	—	1.44	1.23
16	—	1301	211	—	1.45	1.25
18	—	3056	363	—	1.46	1.26
20	—	7276	676	—	1.47	1.27
22	—	18094	1219	—	1.48	1.28
24	—	39135	1641	—	1.48	1.26

(Oc S. Russell & P. Norwig, AIMA)

⇒ Dramatic performance
improvement

Admissible Heuristics from Relaxed Problems

Idea: Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem

- Relaxed 8-puzzle: a tile can move from any tile to any other tile
- $\Rightarrow h_1(n)$ gives the shortest solution

Relaxed 8-puzzle: a tile can move to any adjacent square

$\Rightarrow h_2(n)$ gives the shortest solution



The relaxed problem adds edges to the state space

\Rightarrow any optimal solution in the original problem is also a solution in the relaxed problem

\Rightarrow the cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem

the derived heuristic is an exact cost for the relaxed problem

\Rightarrow must obey the triangular inequality

\Rightarrow consistent

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Inferring Automatically Admissible Heuristics

Idea: If a problem definition is written down in a formal language, it is possible to construct relaxed problems automatically

Example

- 8-puzzle actions:

- we can generate three relaxed problems by removing one or both of the conditions

- (a) a tile can move from square A to square B if A is adjacent to B
- (b) a tile can move from square A to square B if B is blank
- (c) a tile can move from square A to square B

\Rightarrow (a) corresponds to $h_2(n)$, (c) corresponds to $h_1(n)$,

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- 8-puzzle actions:

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 - A is horizontally or vertically adjacent to B, and
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The tool ADGsolver can generate such heuristics automatically

Learning Admissible Heuristics

- Another way to find an admissible heuristic is through **learning from experience**:
 - Experience = solving lots of 8-puzzles
 - An **inductive learning** algorithm can be used to predict costs for other states that arise during search