Chapter 5
Image Restoration

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- Restoration attempts to reconstruct or recover an image that has been degraded by using a priori knowledge of the degradation phenomenon.

Original image

Blurred image

Outline

- A Model of the Image Degradation/Restoration Process
- Noise Models
- Restoration in the Presence of Noise Only-Spatial Filtering
- Periodic Noise Reduction by Frequency Domain Filtering
- Linear, Position-Invariant Degradations
- Estimating the Degradation Function
- Inverse Filtering
- Minimun Mean Square Error(Wiener) Filtering
- Constrained Least Squared Filtering
- Geometric Mean Filter
- Geometric Transformations

- If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * F(x, y) + \eta(x, y)$$

- Where h(x,y) is the spatial representation of the degradation function.
- Write the model in an equivalent frequency domain representation.

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

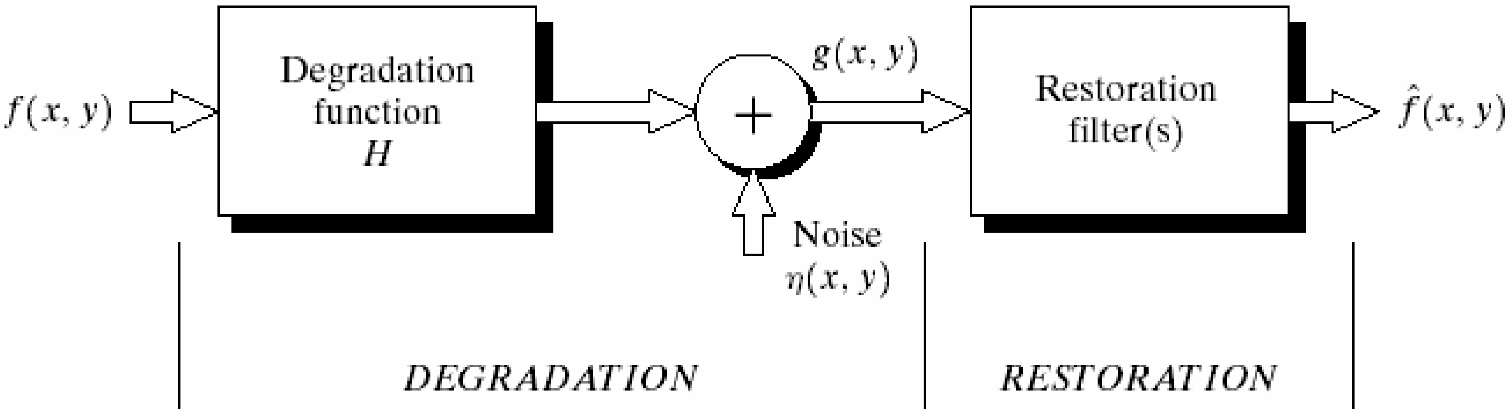

FIGURE 5.1 A model of the image degradation/ restoration process.

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- Spatial and Frequency Properties of Noise
- Some Importany Noise Probability Density Functions
- Periodic Noise
- Estimation of Noise Parameters

- Parameters: define the spatial characteristics of noise, and whether the noise is correlated with the image.
- Frequency properties: refer to the frequency content of noise in the Fourier sense.
- When the Fourier spectrum of noise is constant, the noise usually is called white noise.

- Spatial and Frequency Properties of Noise
- Some Importany Noise Probability Density Functions
- Periodic Noise
- Estimation of Noise Parameters

Gaussian noise

- The PDF of a Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

- z represents gray level,
- μ is the mean of average value of z;
- σ is its standard deviation.

Rayleigh noise

- The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for} \quad z \geq a \\ 0 & \text{for} \quad z < a \end{cases}$$

- The mean and variance of this density are given by

$$\mu = a + \sqrt{\pi b / 4}$$

- and

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Erlang(Gamma) noise

- The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & for \quad z \geq 0 \\ 0 & for \quad z < 0 \end{cases}$$

- The mean and variance of this density are given by

$$\mu = \frac{b}{a}$$

- and

$$\sigma^2 = \frac{b}{a^2}$$

Exponential noise

- The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & for \quad z \geq 0 \\ 0 & for \quad z < 0 \end{cases}$$

- The mean and variance of this density function are

$$\mu = \frac{1}{a}$$

- and

$$\sigma^2 = \frac{1}{a^2}$$

Uniform noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & if \quad a \le z \le b \\ 0 & otherwise \end{cases}$$

The mean of this density function is given by

$$\mu = \frac{a+b}{2}$$

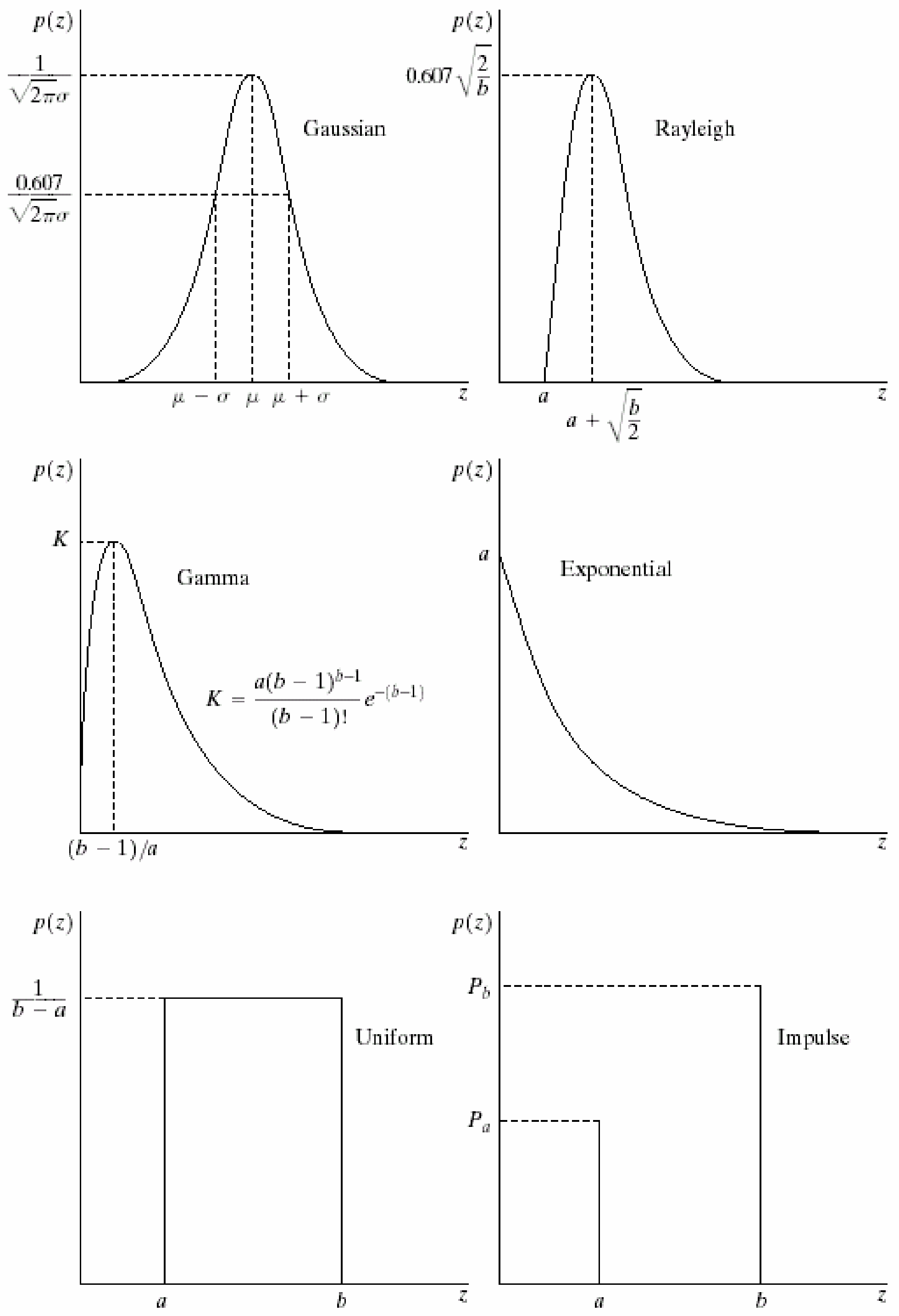
And its variance by

$$\sigma^2 = \frac{(b-a)^2}{12}$$

Impulse (salt-and-pepper) noise

- The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for} \quad z = a \\ P_b & \text{for} \quad z = b \\ 0 & \text{otherwise} \end{cases}$$



a	b
c	d
e	f

FIGURE 5.2 Some important probability density functions.

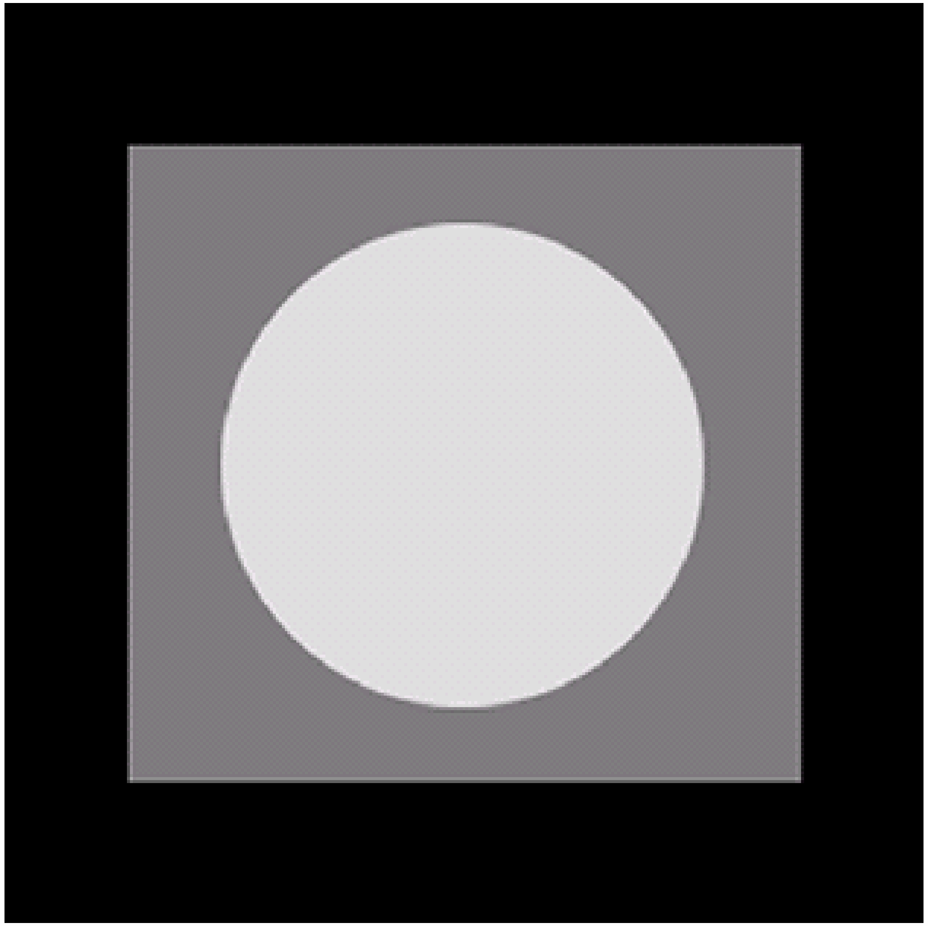


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

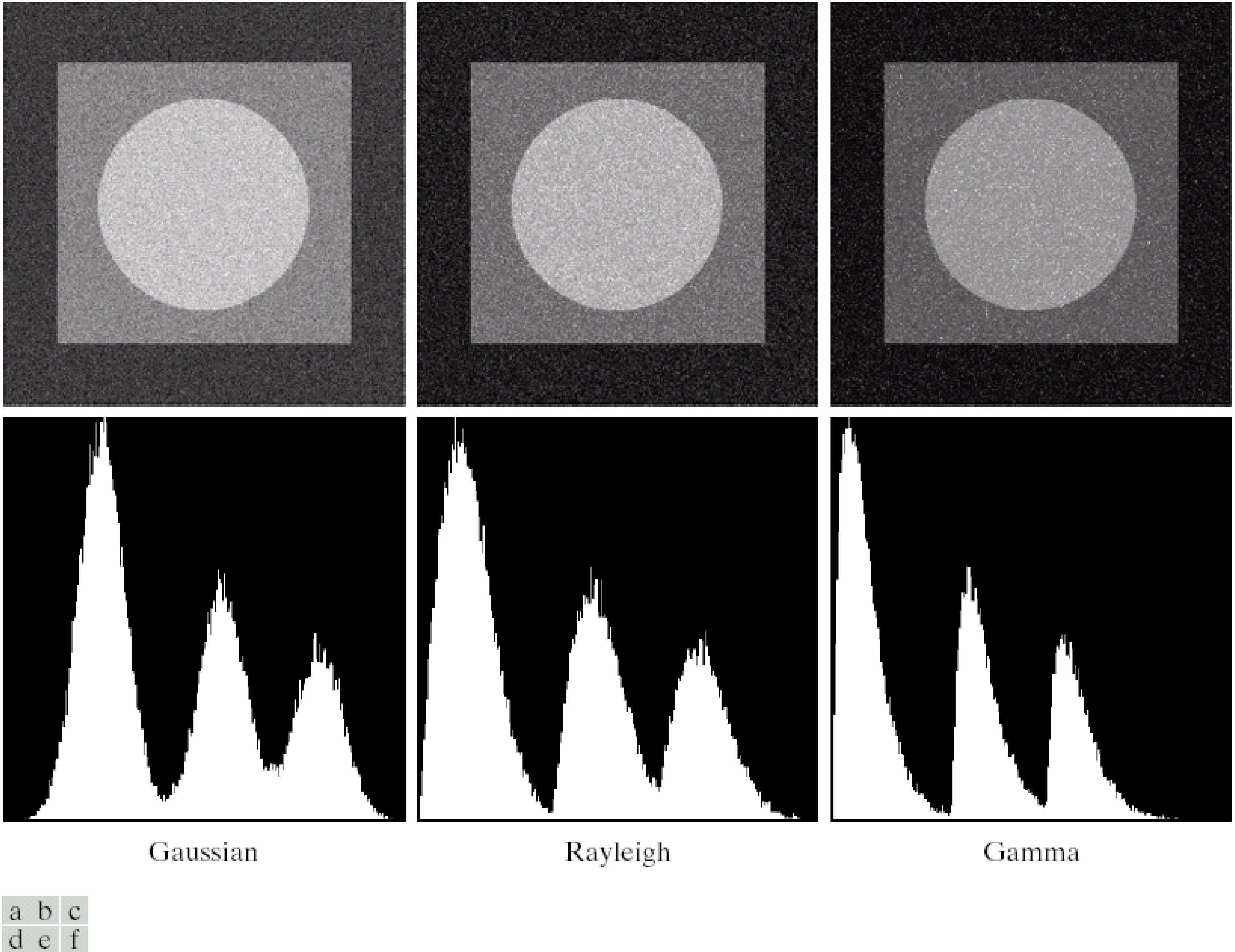


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

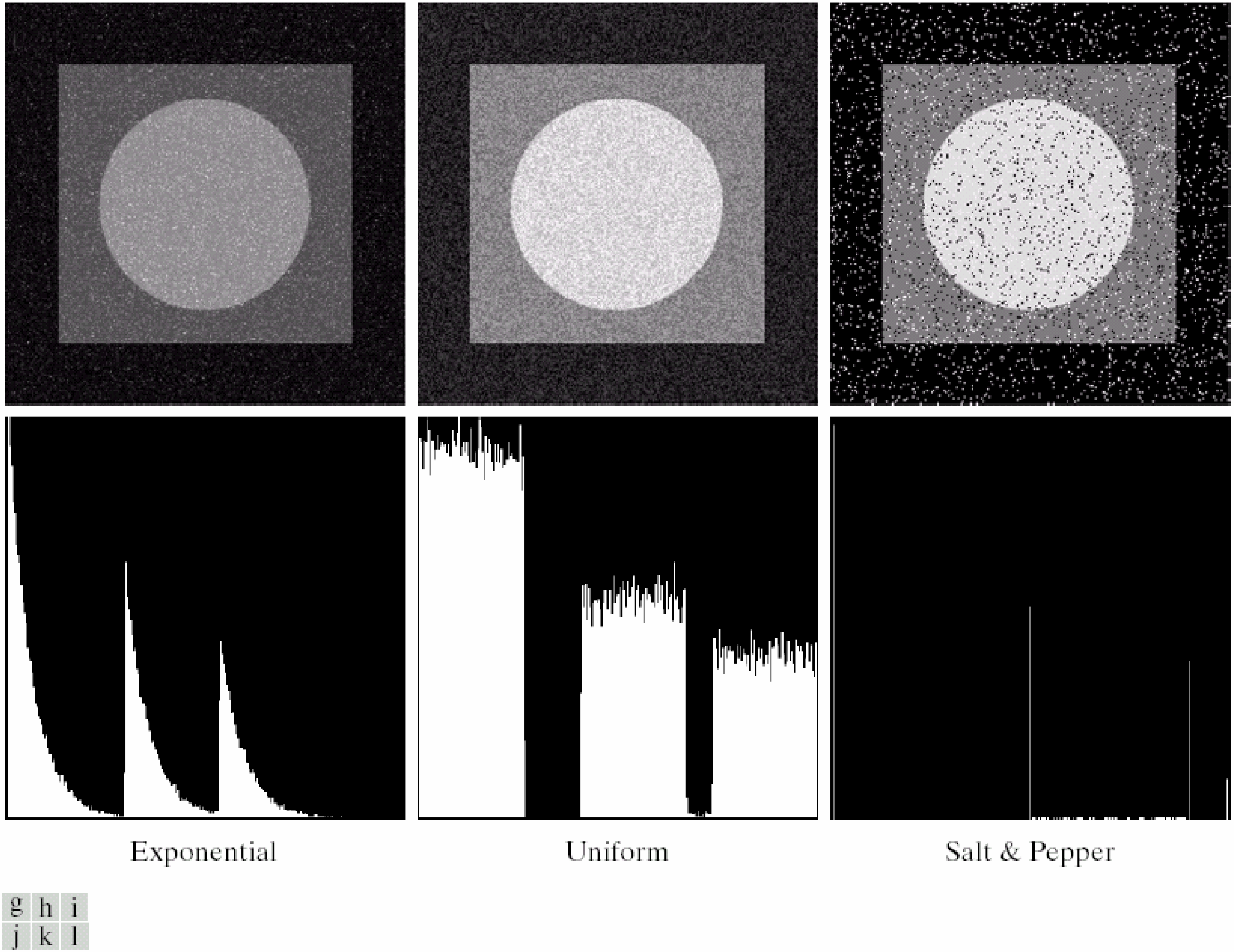


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

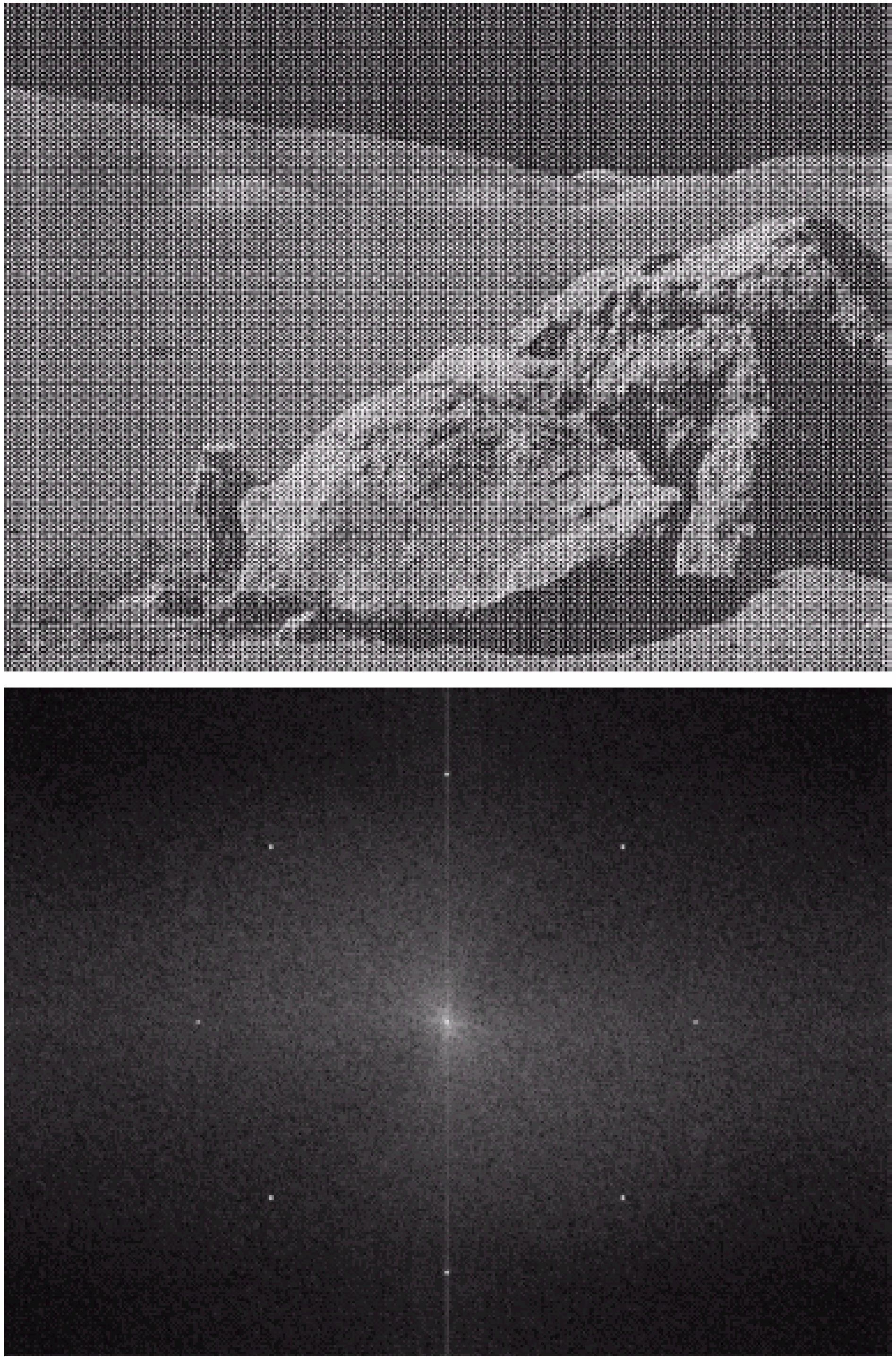
- Spatial and Frequency Properties of Noise
- Some Importany Noise Probability Density Functions
- Periodic Noise
- Estimation of Noise Parameters

- Periodic noise in an image irises typically from electrical or electromechanical interference during image acquisition.

- Periodic noise can be reduced significantly via frequency domain filtering.

a
b

FIGURE 5.5
(a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)



- Spatial and Frequency Properties of Noise
- Some Importany Noise Probability Density Functions
- Periodic Noise
- Estimation of Noise Parameters

- The parameters of periodic noise typically are estimated by inspection of the Fourier spectrum of the image.
- The simplest way to use the data from the image strips is for calculating the mean and variance of the gray levels.

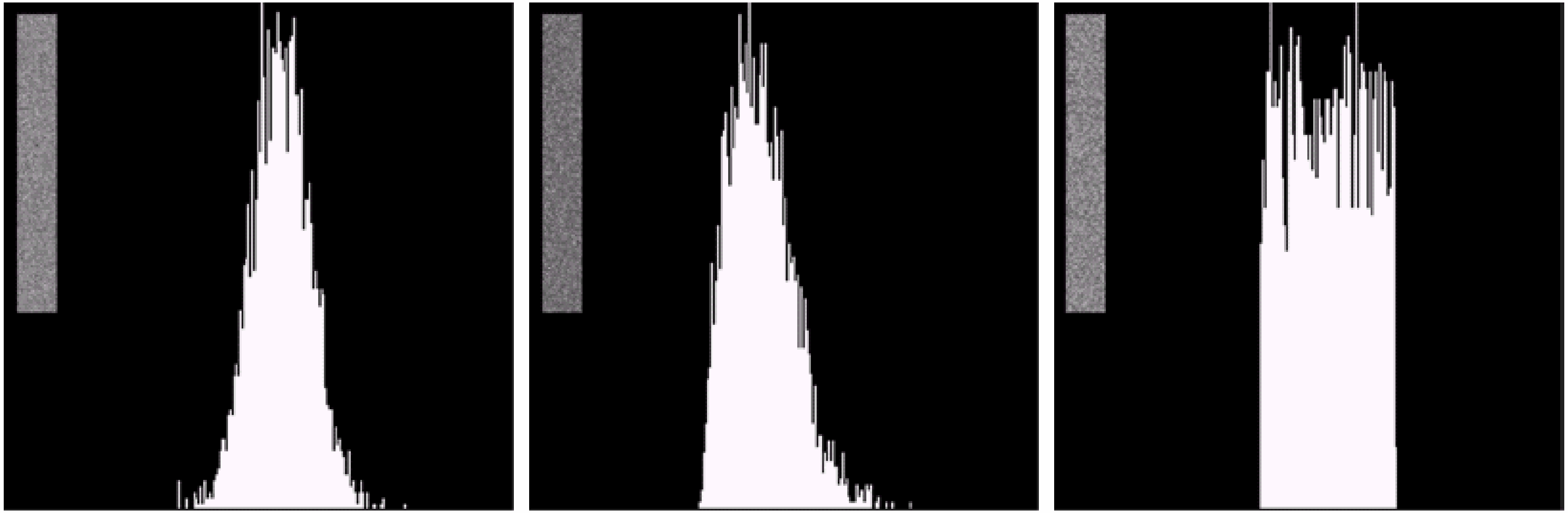
- Consider a strip (subimage) denoted by S. We can use the following sample approximations from basic statistics:

$$\mu = \sum_{z_i \in S} z_i p(z_i)$$

and

$$\sigma^2 = \sum_{z_i \in S} (z_i - u)^2 p(z_i)$$

where the zi's are the gray-level values of the pixels in S, and $p(z_i)$ are the corresponding normalized histogram values.



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

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- When the only degradation present in an image is noise

$$g(x, y) = f(x, y) + \eta(x, y)$$

and

$$G(u, v) = F(u, v) + N(u, v)$$

- Mean Filters
- Order-Statistics Filters
- Adaptive Filters

Arithmetic mean filter

- Let S_{xy} represent the set of coordinates in a rectangular subimage window of size m*n, centered at point (x,y).
- The arithmetic mean filtering process computes the average value of the corrupted image g(x,y) in the area defined by Sxy.
- The value of the restored image $\hat{f}$ at any point (x,y) is simply the arithmetic mean computed using the pixels in the region defined by S_{xy}. In other words,

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Geometric mean filter

- An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Harmonic mean filter

- The harmonic mean filtering operation is given by the expression

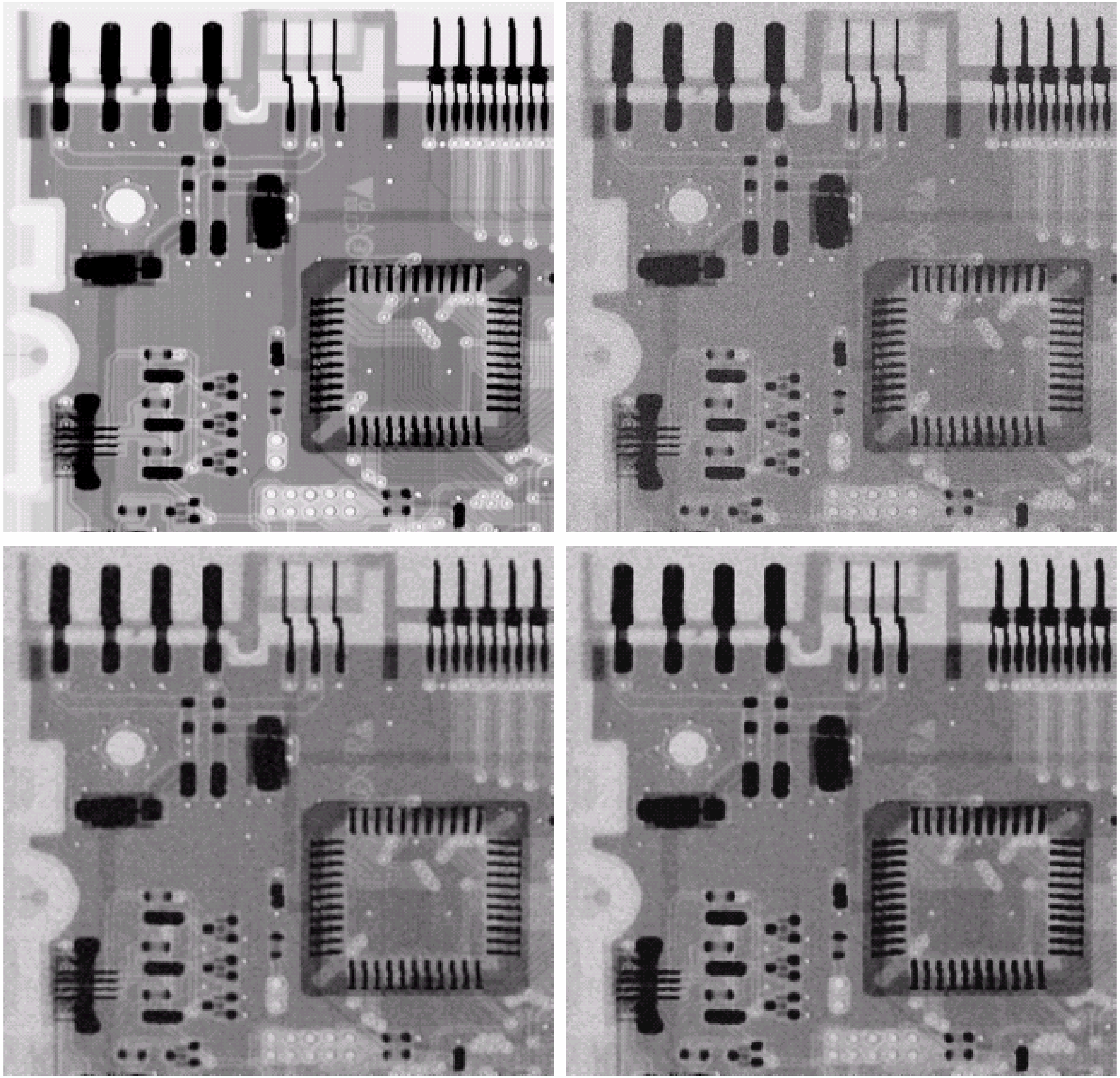
$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Contraharmonic mean filter

- The contraharmonic mean filtering operation yields a restored image based on the expression:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

where Q is called the order of the filter.



a	b
c	d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

a	b
c	d

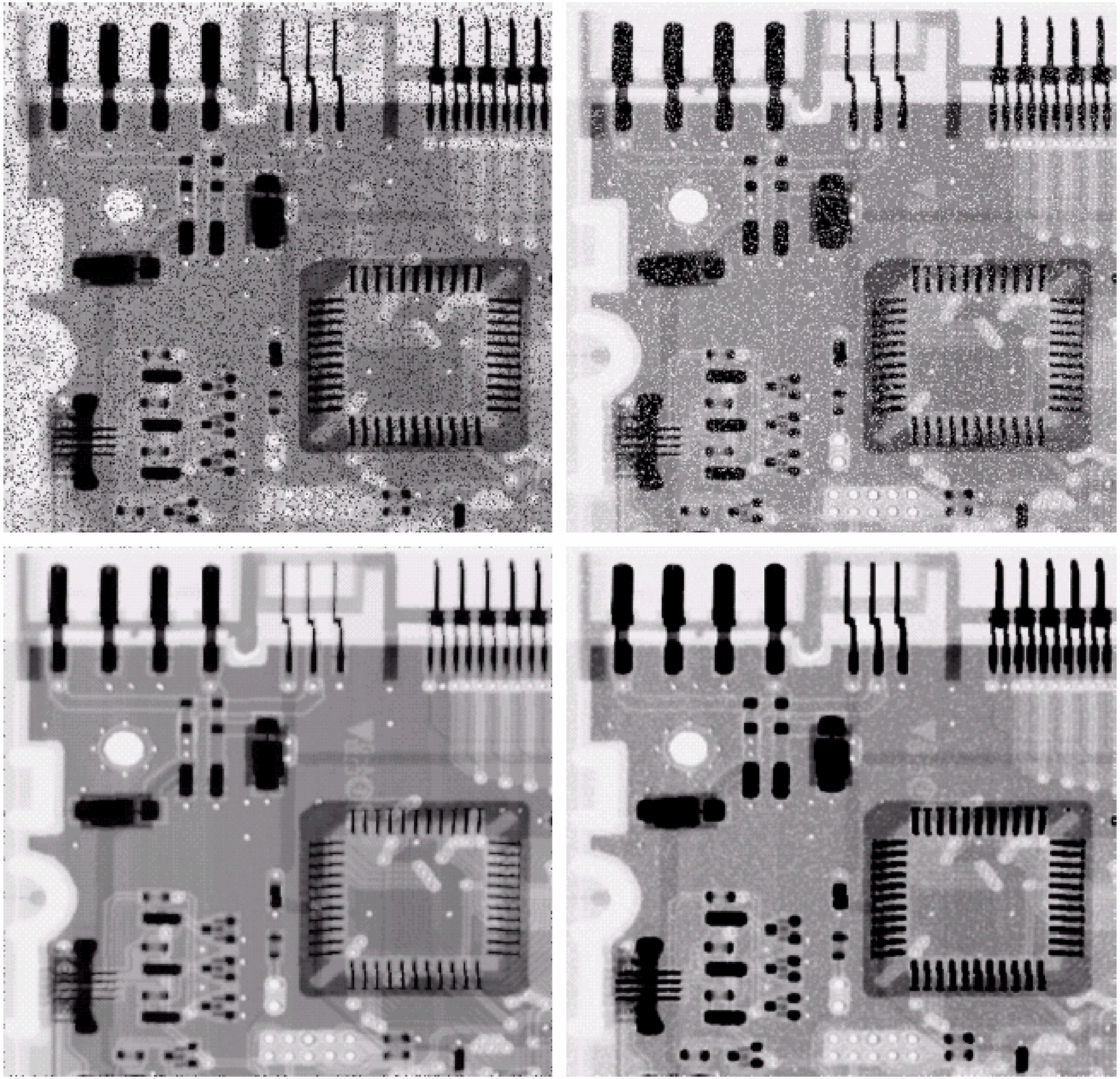
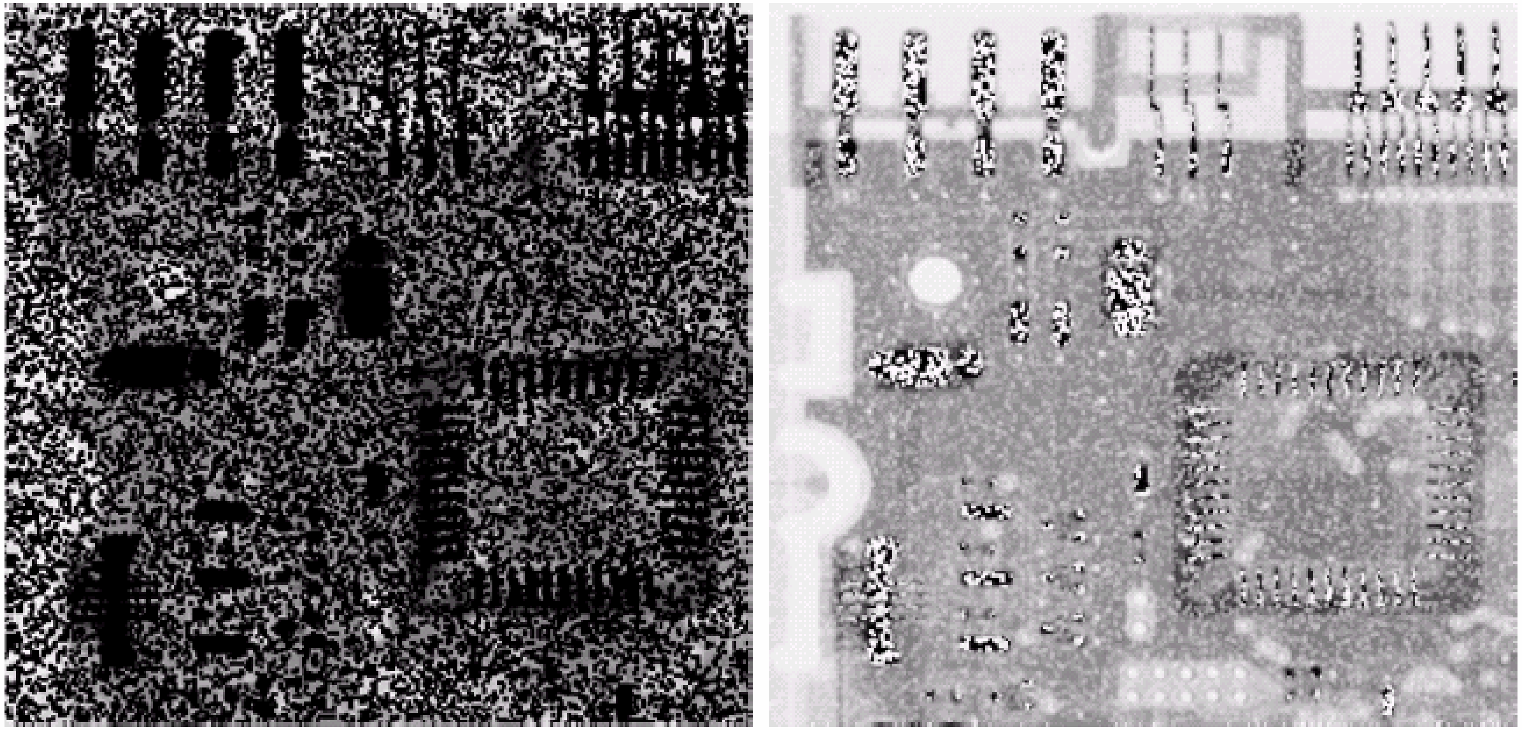


FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



a b

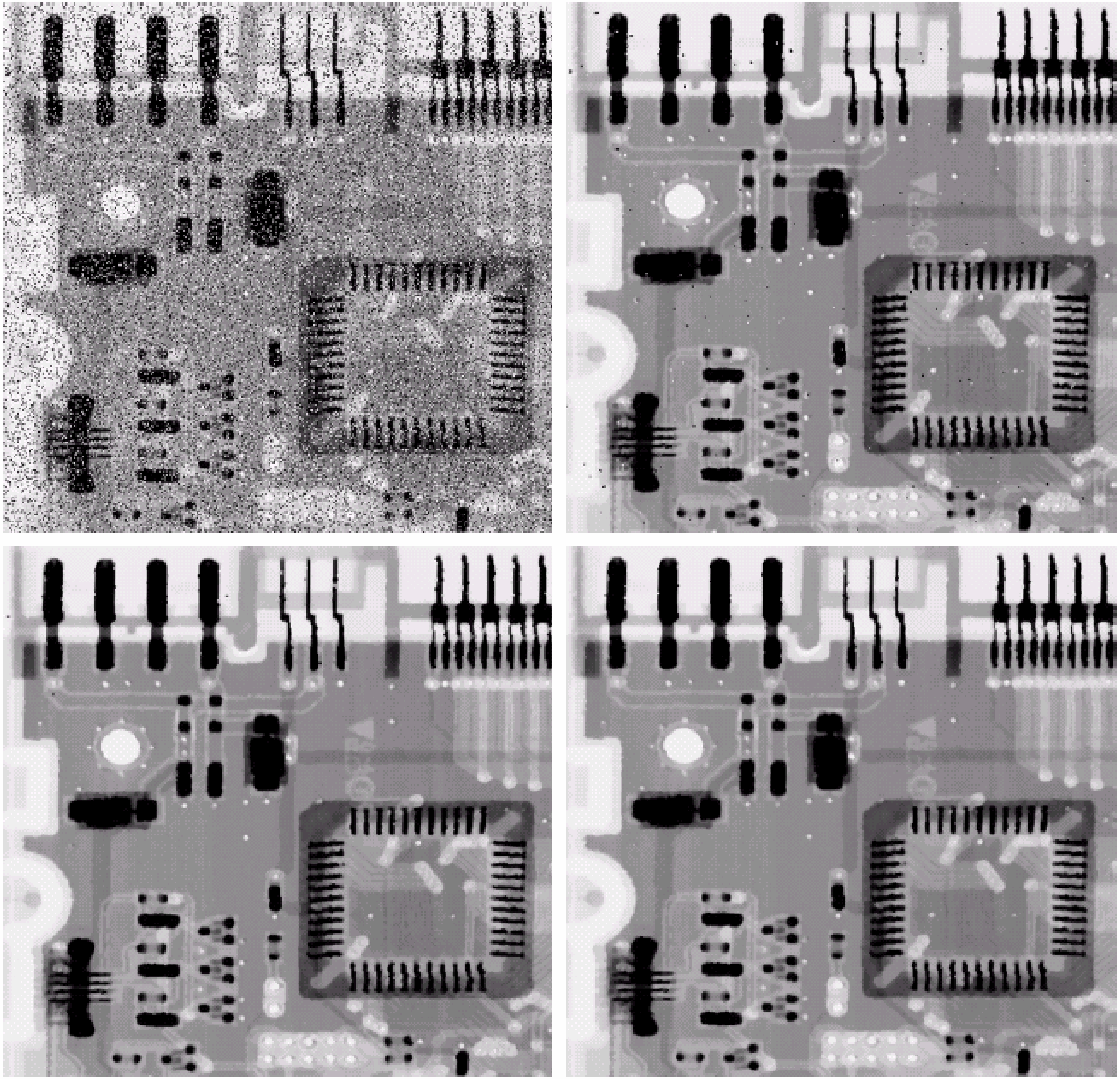
FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

- Mean Filters
- Order-Statistics Filters
- Adaptive Filters

Median filter

- Replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel:

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

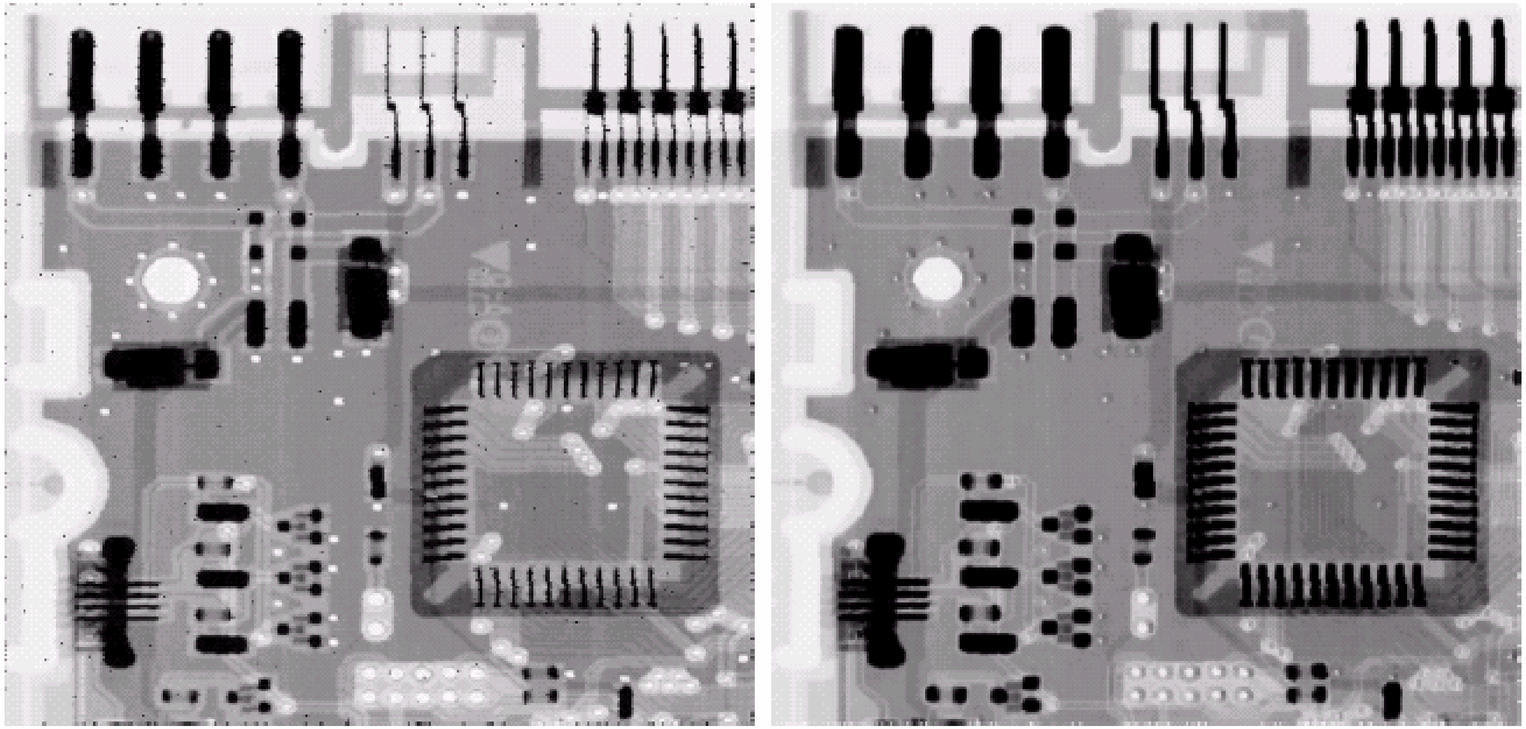
Max and min filters

- Using the 100th percentile results in the so-called max filter, given by

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- The 0th percentile filter is the min filter:

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$



a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Midpoint filter

- The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter:

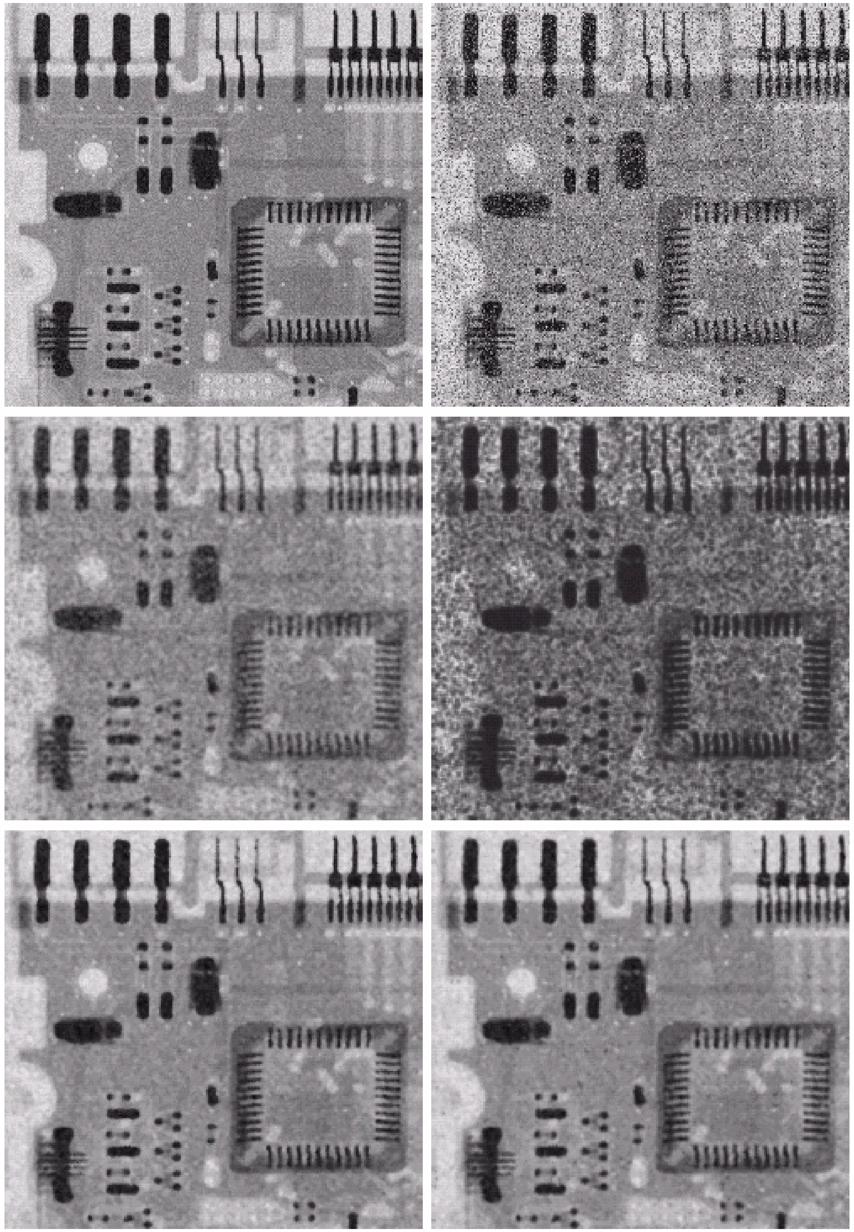
$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Alpha-trimmed mean filter

- Suppose that we delete the d/2 lowest and the d/2 highest gray-level values of g(s,t) in the neighborhood S_{xy}.Let $g_r(s,t)$ represent the remaining mn-d pixels. A filter formed by averaging these remaining pixels is called an alpha-trimmed mean filter:

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

- Where the value of d can range from 0 to mn-1.
- When d=0, the alpha-trimmed filter reduces to the arithmetic mean filter.
- If we choose d=mn-1, the filter becomes a median filter.



a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5×5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

- Mean Filters
- Order-Statistics Filters
- Adaptive Filters

- Two simple adaptive filters whose behavior changes based on statistical characteristics of the image inside the filter region defined by the m*n rectangular window S_{xy}.

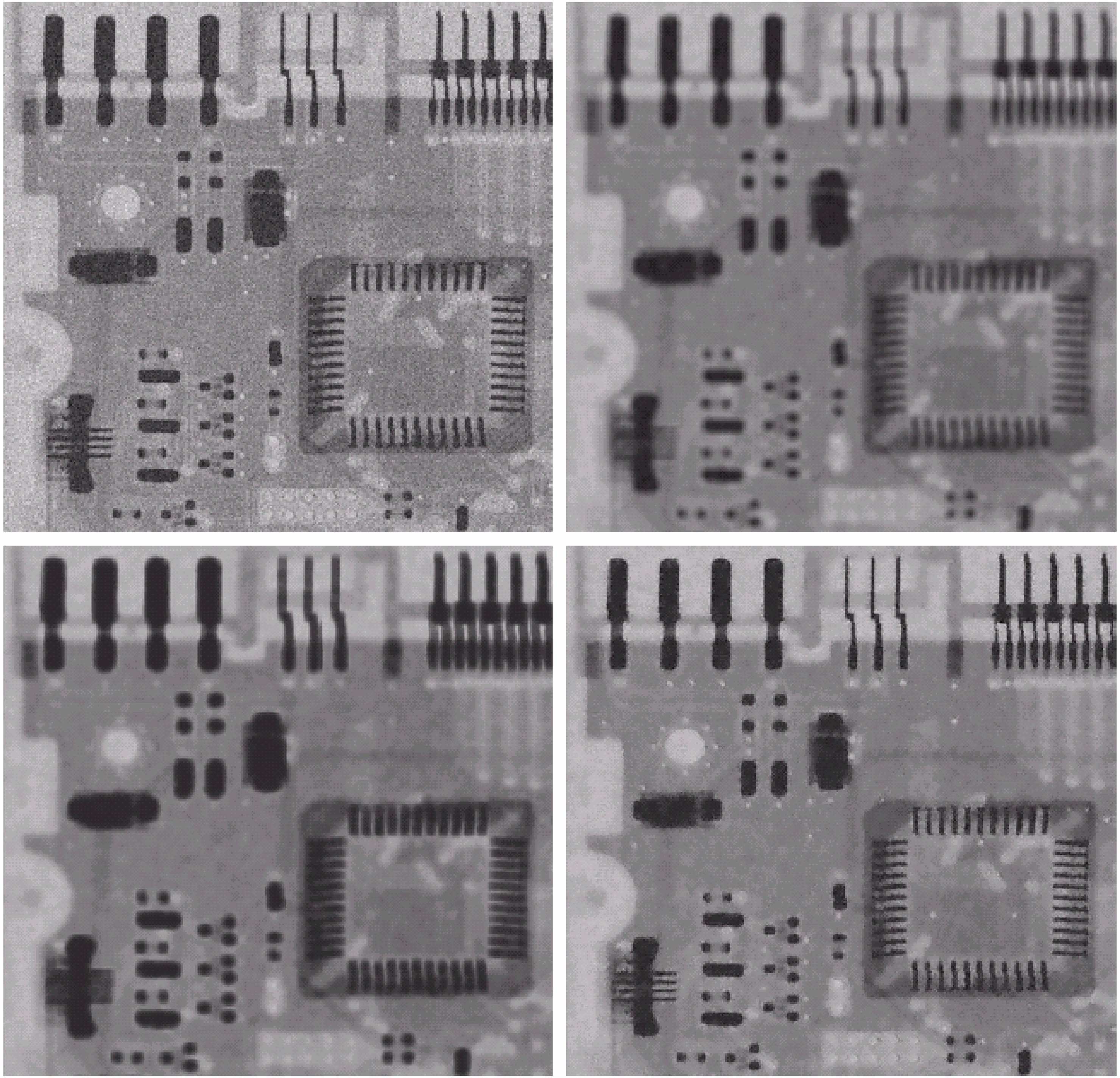
Adaptive, local noise reduction filter

- The response of the filter at any point(x,y) on which the region is centered is to be based on four quantities:
  - g(x,y), the value of the noisy image at (x,y);
  - σ_η^2 , the variance of the noise corrupting f(x,y) to form g(x,y);
  - m_L , the local mean of the pixels in S_{xy};
  - σ_L^2 , the local variance of the pixels in S_{xy}.

- We want the behavior of the filter to be as follows:
  - If σ_η^2 is zero, the filter should return simply the value of g(x, y). This is the trivial, zero-noise case in wnich g(x, v) is equal to f(x, y).
  - If the local variance is high relative to σ_η^2, the filter should return a value close to g(x,y). A nigh local variance typically is associated with edges, and these should be preserved.
  - If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in Sxy. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced simply by averaging.

- An adaptive expression for obtaining $\widehat{f}(x, y)$ based on these assumptions may be written as

$$\widehat{f}(x, y) = g(x, y) - \frac{\sigma_{\eta}^{2}}{\sigma_{L}^{2}}\left[g(x, y) - m_{L}\right]$$



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

z_{min} = minimum gray level value in S_{xy}

z_{max} = maximum gray level value in S_{xy}

z_{med} = median of gray levels in S_{xy}

z_{xy} = gray level at coordinates (x, y)

S_{max} = maximum allowed size of S_{xy}.

Level A:

$A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If $A1 > 0$ AND $A2 < 0$, Go to level B

Else increase the window size

If window size $\leq S_{max}$ repeat level A

Else output z_{med}.

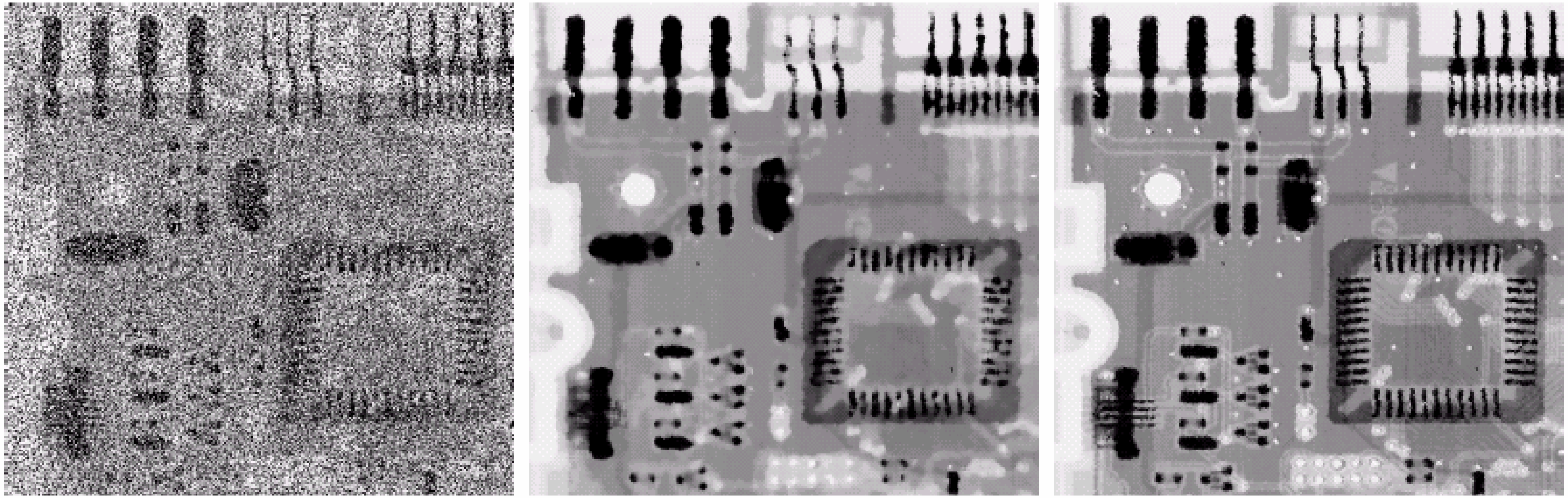
Level B:

$B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If $B1 > 0$ AND $B2 < 0$, output z_{xy}

Else output z_{med}.



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

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- Bandreject Filters
- Bandpass Filters
- Notch Filters
- Optimum Notch Filtering

- Bandreject filters remove or attenuate a band of frequencies about the origin of the Fourier transform. An ideal bandreject filter is given by the expression

$$H(u,v) = \begin{cases} 1 & \text{if} \quad D(u,v) < D_0 - \frac{W}{2} \\ 0 & \text{if} \quad D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & \text{if} \quad D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

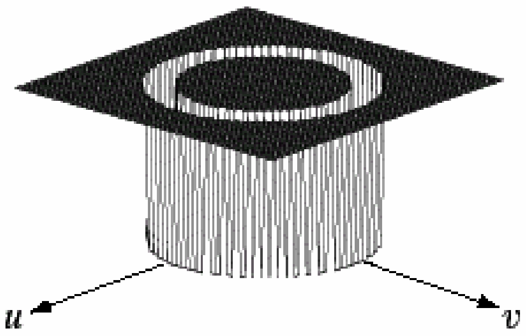
- Where D(u,v) is the distance from the origin of the centered frequency rectangle, W is the width of the band, and D0 is its radial center.

- A Butterworth bandreject filter of order n is given by the expression:

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)W}{D^2(u,v) - D_0^2} \right]^{2n}}$$

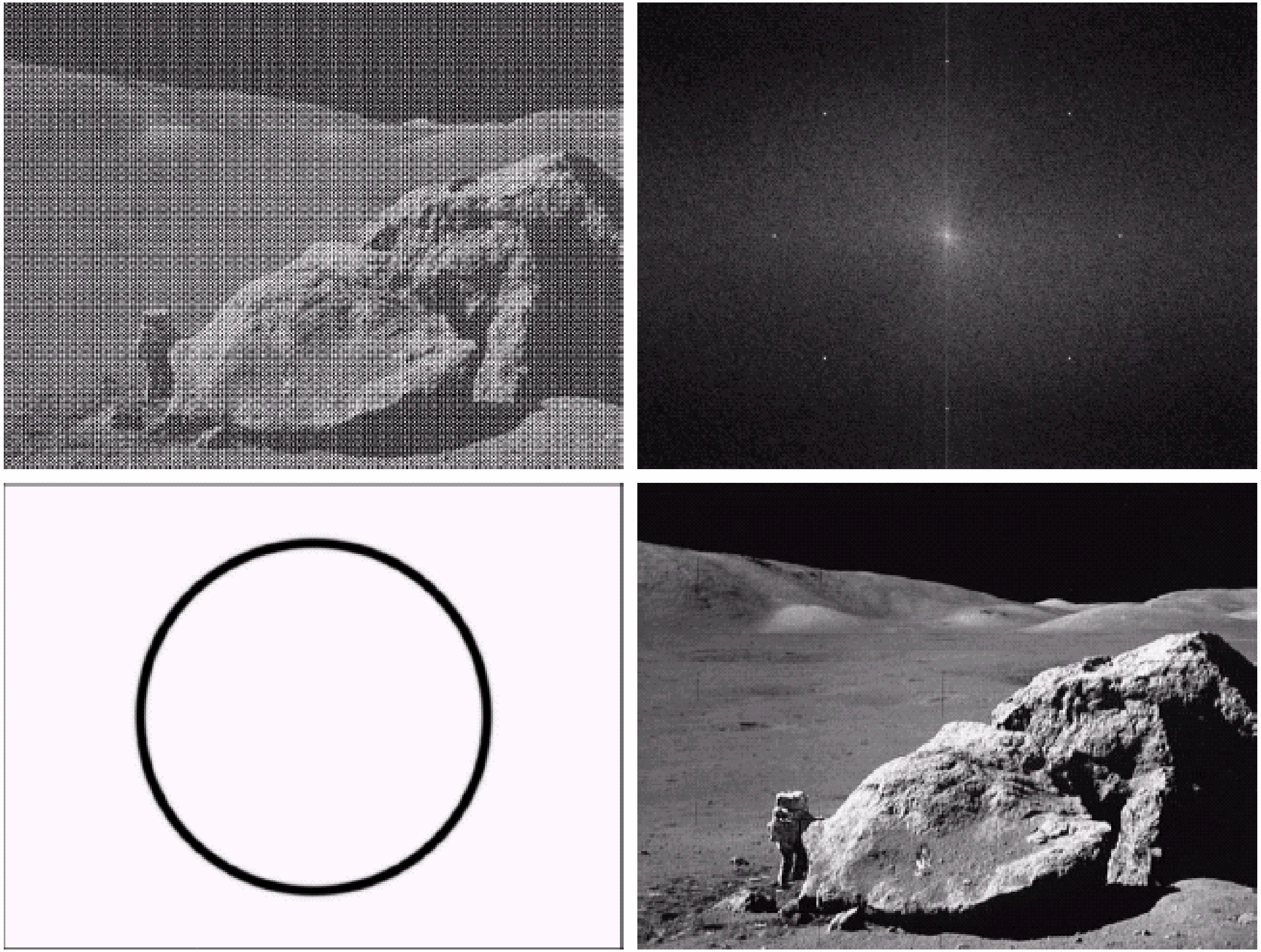
- And a Gaussian bandreject filter is given by

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)W} \right]}$$



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a	b
c	d

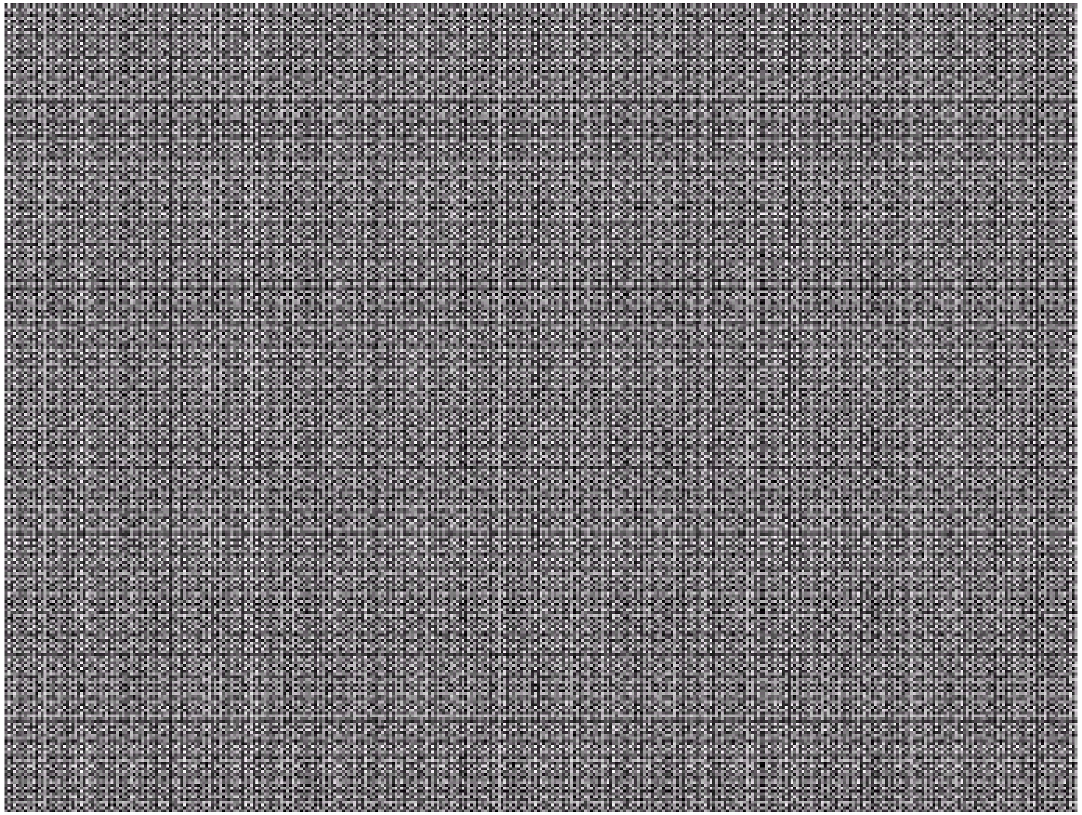
FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

- Bandreject Filters
- Bandpass Filters
- Notch Filters
- Optimum Notch Filtering

- The transfer function $H_{bp}(u,v)$ of a bandpass filter obtained from a corresponding bandreject filter with transfer function $H_{br}(u,v)$ by using the equation

$$H_{bp}(u,v)=1-H_{br}(u,v)$$

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



- Bandreject Filters
- Bandpass Filters
- Notch Filters
- Optimum Notch Filtering

- The transfer function of an ideal notch reject filter of radius D0, with centers at(u0,v0) and, by symmetry, at(-u0,-v0), is

$$H(u,v) = \begin{cases} 0 & \text{if} \quad D(u,v) \le D_0 \quad \text{or} \quad D_2(u,v) \le D_0 \\ 1 & \text{otherwise} \end{cases}$$

where

$$D_1(u,v) = \left[(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2 \right]^{1/2}$$

and

$$D_2(u,v) = \left[(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2 \right]^{1/2}$$

- The transfer function of a Butterworth notch reject filter of order n is given by

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v) D_2(u,v)} \right]^n}$$

- A Gaussian notch reject filter has the form

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v) D_2(u,v)}{D_0^2} \right]}$$

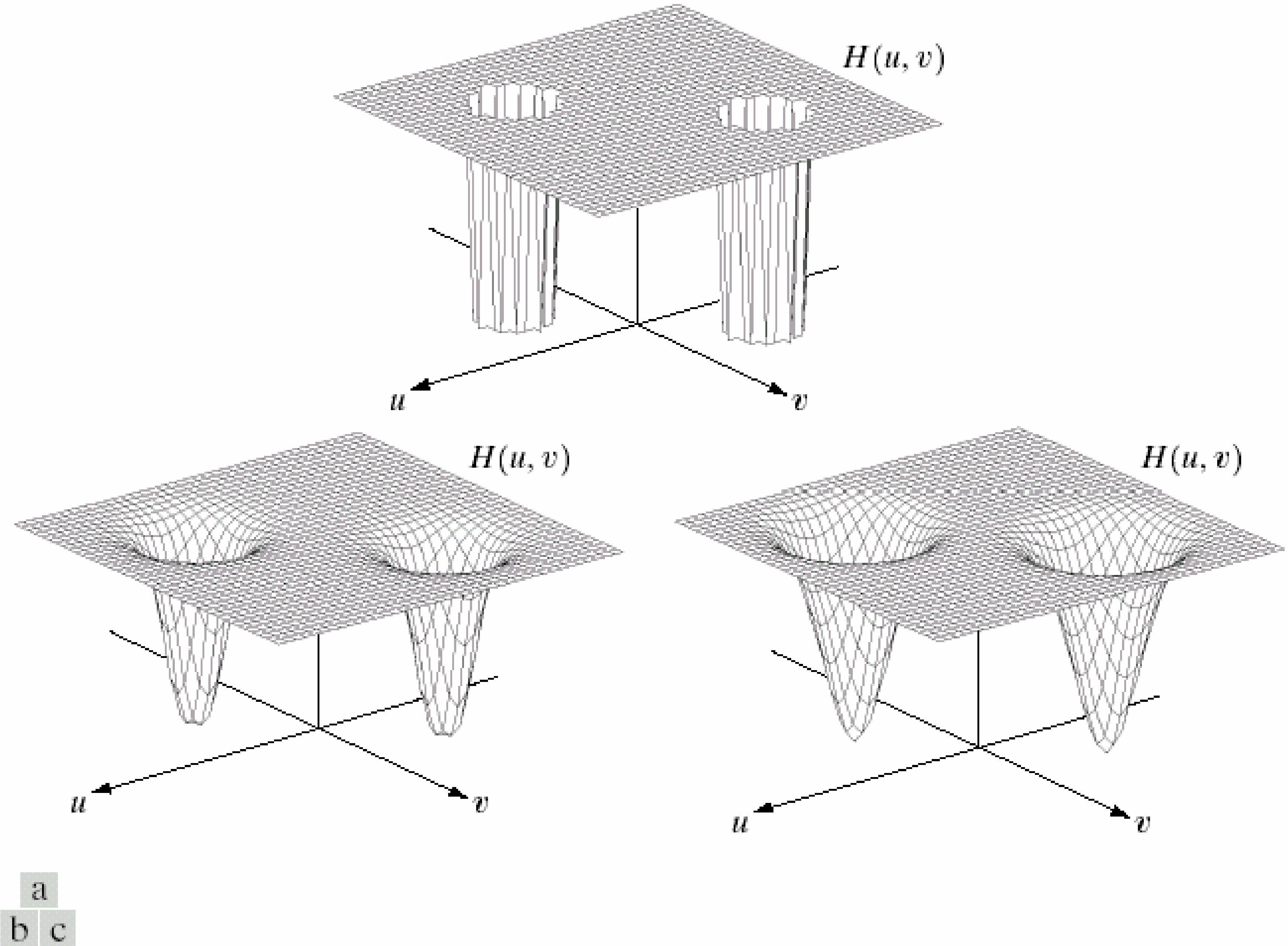


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

- Bandreject Filters
- Bandpass Filters
- Notch Filters
- Optimum Notch Filtering

- The Fourier transform of the interference noise pattern is given by the expression

$$N(u,v) = H(u,v)G(u,v)$$

- The corresponding pattern in the spatial domain is obtained from the expression

$$\eta(x,y) = \mathfrak{I}^{-1}\left\{H(u,v)G(u,v)\right\}$$

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

- The local variance of $\hat{\mathrm{f}}(\mathrm{x}, \mathrm{y})$ at coordinates (x,y) can be estimated from the samples as follows:

$$\sigma^2(\mathrm{x}, \mathrm{y}) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x, y) \right]^2$$

- Where $\bar{\hat{f}}(x, y)$ is the average value of $\hat{f}$ in the neighborhood

$$\bar{\hat{f}}(x, y) = \frac{1}{(2\mathrm{a}+1)(2\mathrm{b}+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \hat{f}(x+s, y+t)$$

$$\sigma^2(\mathrm{x},\mathrm{y}) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{[g(x+s, y+t) - w(x,y)\eta(x+s, y+t)]$$

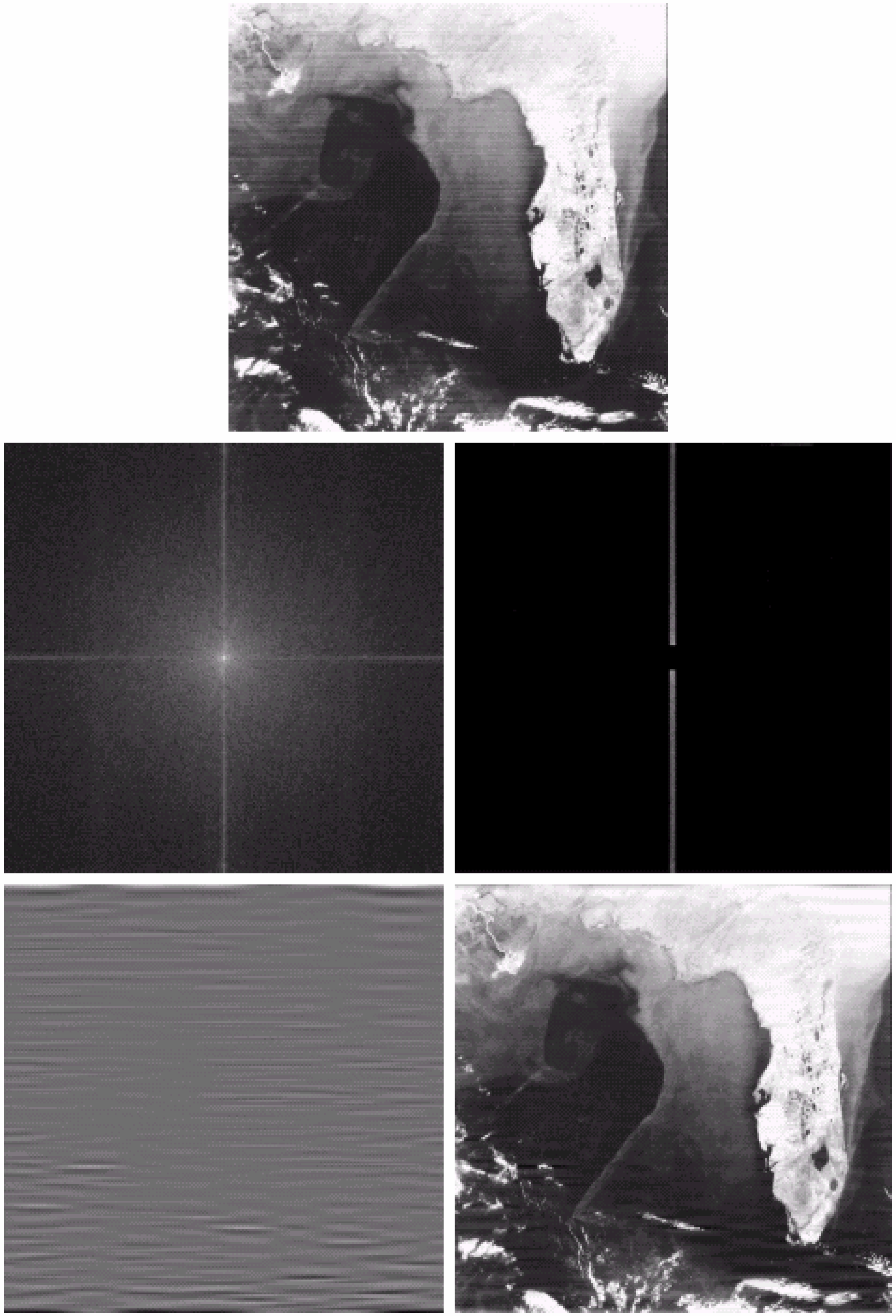
$$- w(x,y)\eta(x+s, y+t)] - [\bar{g}(x,y) - w(x,y)\bar{\eta}(x,y)]\}^2$$

- To minimize $\sigma^2(x,y)$,we solve

$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$$

for w(x,y). The result is

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

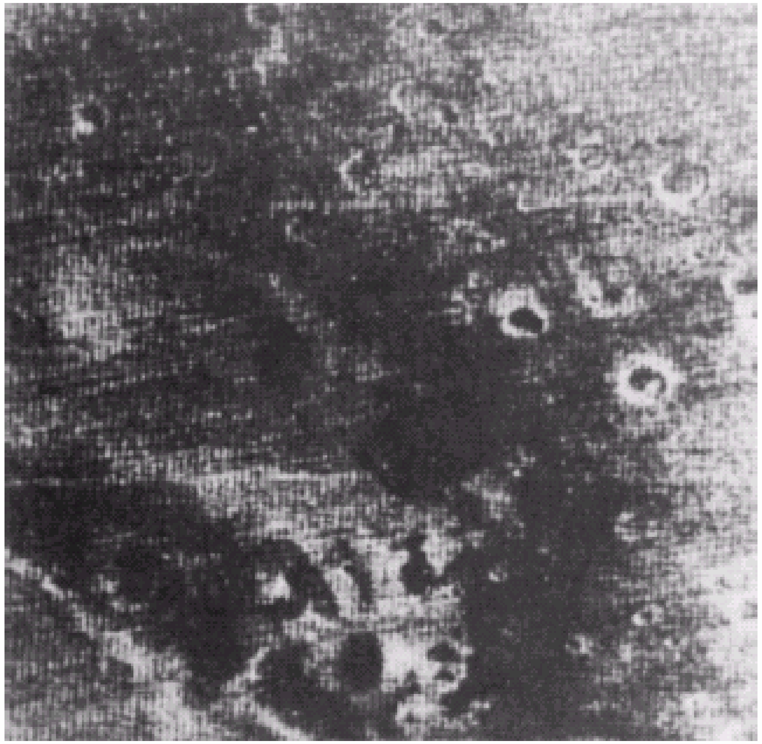


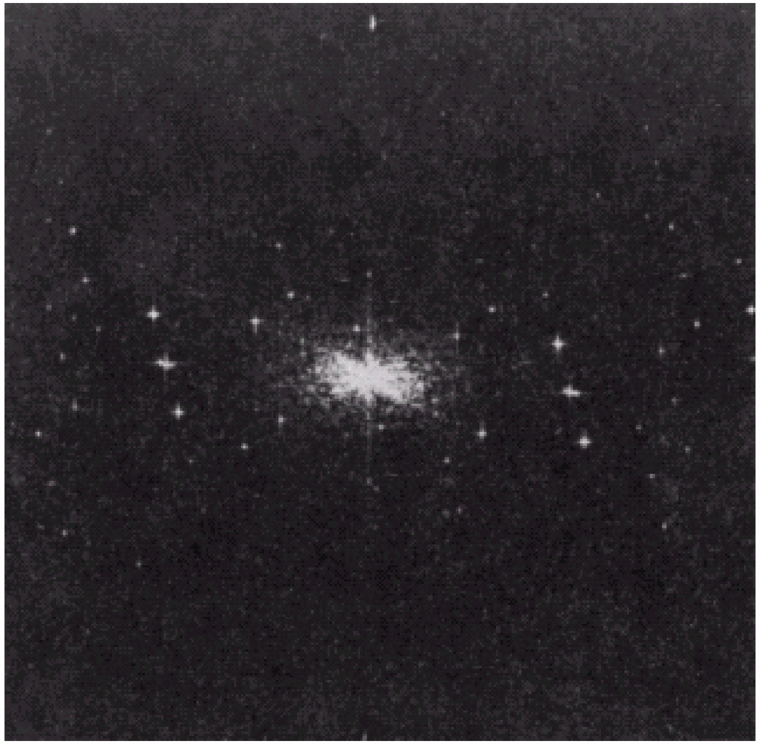
a
b c
d e

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico (note horizontal sensor scan lines). (b) Spectrum of (a). (c) Notch pass filter shown superimposed on (b). (d) Inverse Fourier transform of filtered image, showing noise pattern in the spatial domain. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

a b

FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6.*
(b) Fourier spectrum showing periodic interference.
(Courtesy of NASA.)





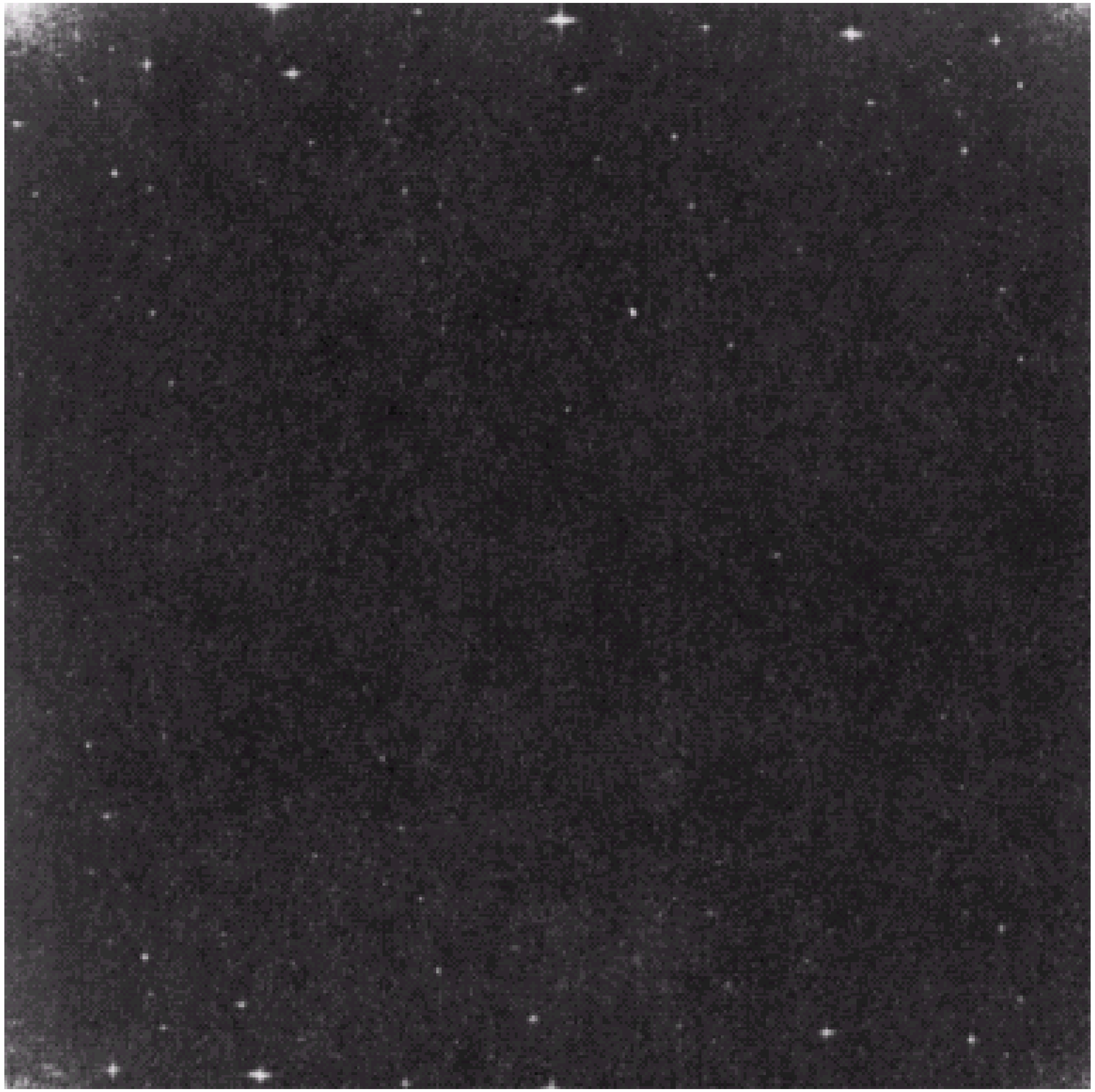
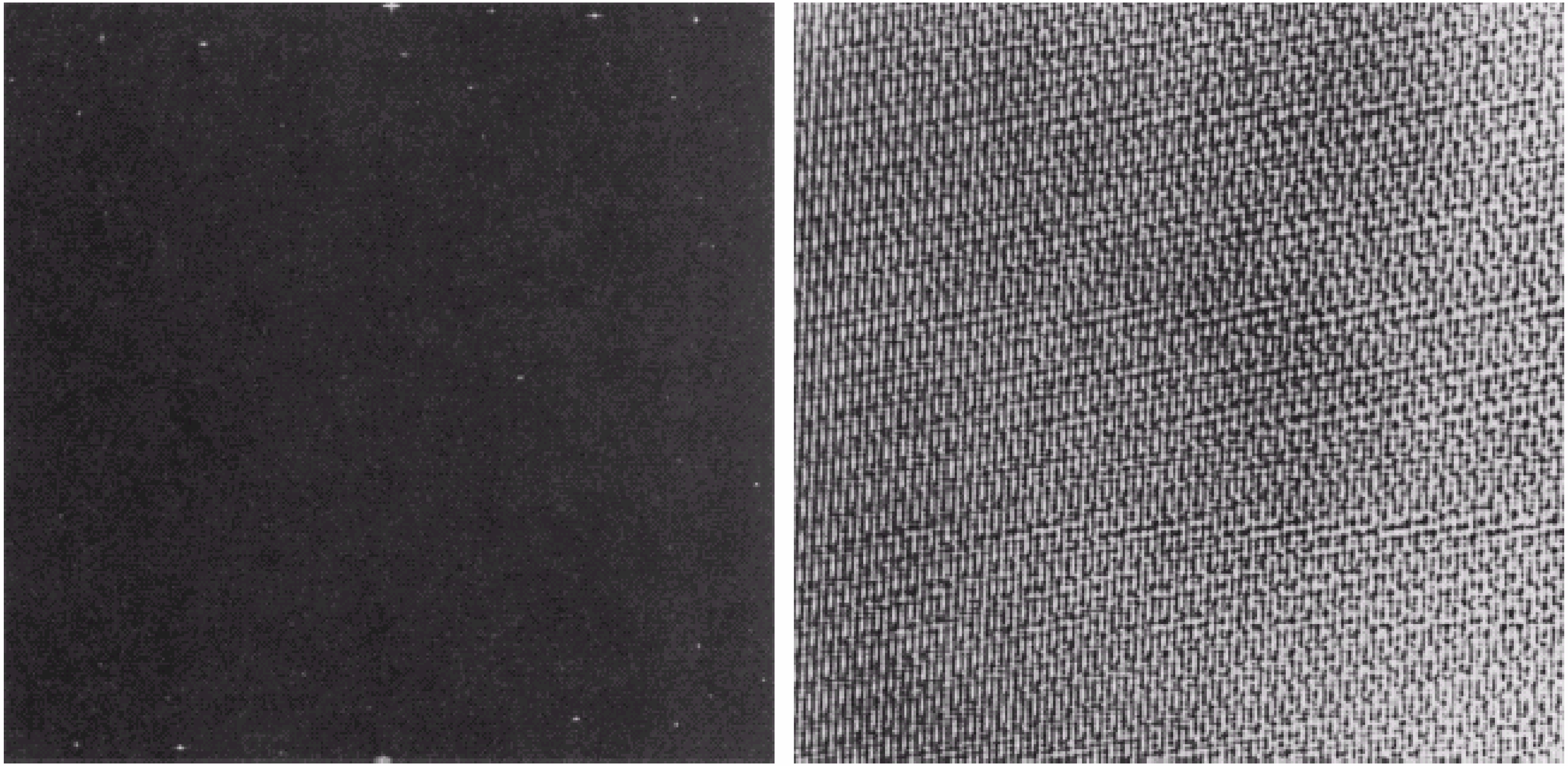


FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)



a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

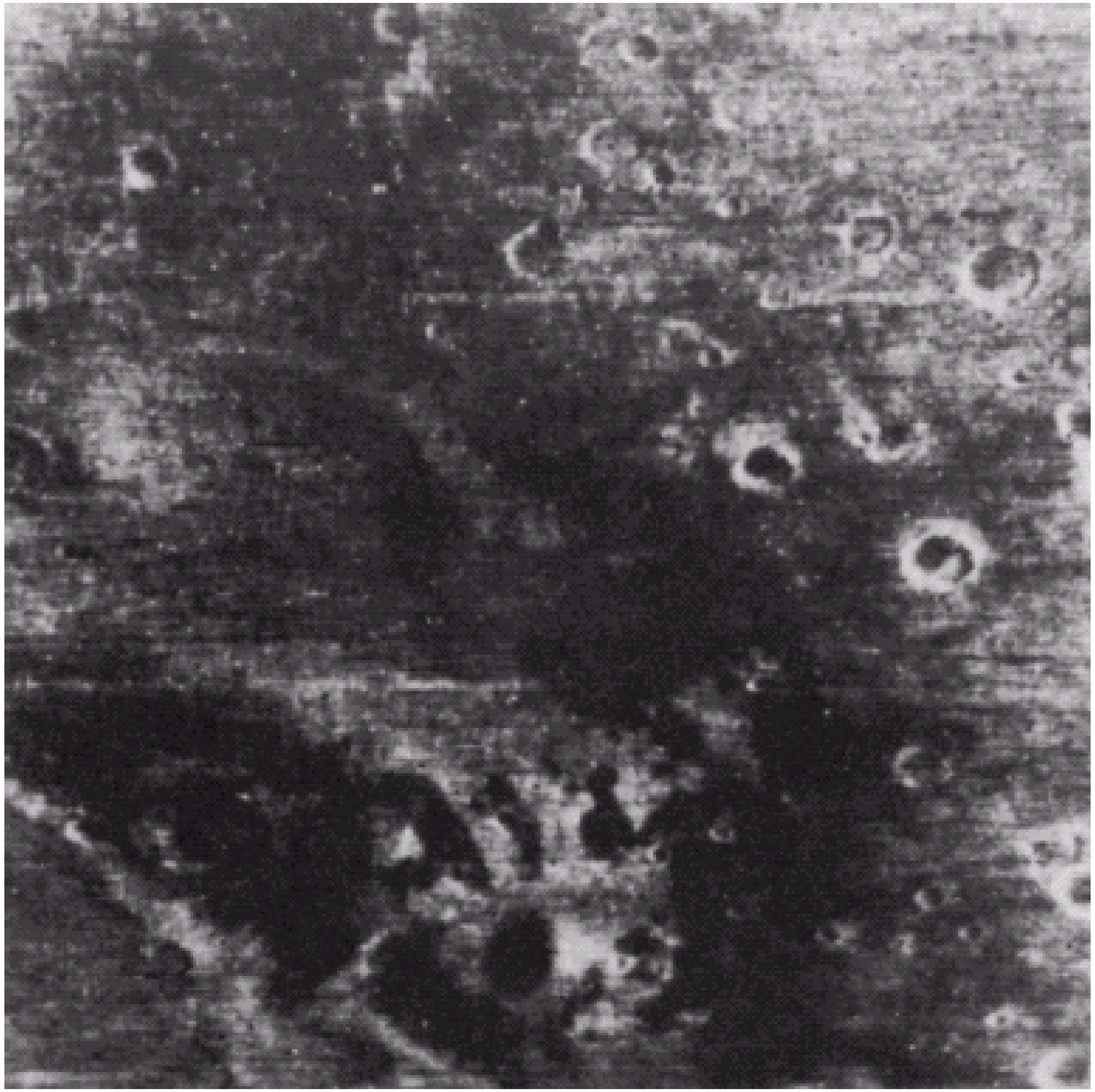


FIGURE 5.23 Processed image. (Courtesy of NASA.)

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- The input-output relationship is expressed as

$$g(x,y) = H\left[f(x,y)\right] + \eta(x,y)$$

$$\eta(x,y) = 0$$

- H is linear if

$$H\left[af_1(x,y) + bf_2(x,y)\right] = aH\left[f_1(x,y)\right] + bH\left[f_2(x,y)\right]$$

- An operator having the input-output relationship $g(x,y) = H\left[f(x,y)\right]$ is said to be position invariant if

$$H\left[f(x-a, y-b)\right] = g(x-a, y-b)$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\,d\alpha\, d\beta$$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)d\alpha d\beta$$

- Where $h(x,\alpha,y,\beta)=H\left[\delta(x-\alpha,y-\beta)\right]$
- If H is position invariant

then, $H\left[\delta(x-\alpha,y-\beta)\right]=h(x-\alpha,y-\beta)$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta$$

This expression is called the convolution integral.

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- Estimation by Image Observation
- Estimation by Experimentation
- Estimation by Modeling

$$H_s(u,v) = \frac{G_s(u,v)}{\widehat{F}_s(u,v)}$$

- $g_s(x,y)$:The observed subimage;
- $\widehat{f}_s(x,y)$:The constructed subimage;

- Estimation by Image Observation
- Estimation by Experimentation
- Estimation by Modeling

$$H(u,v) = \frac{G(u,v)}{A}$$

- G(u,v): The Fourier transform of the observed image;
- A: a constant describing the strength of the impulse;

- Estimation by Image Observation
- Estimation by Experimentation
- Estimation by Modeling

- A degradation model proposed by Hufnagel and Stanley is based on the physical characteristics of atmospheric turbulence. This model has a familiar form:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

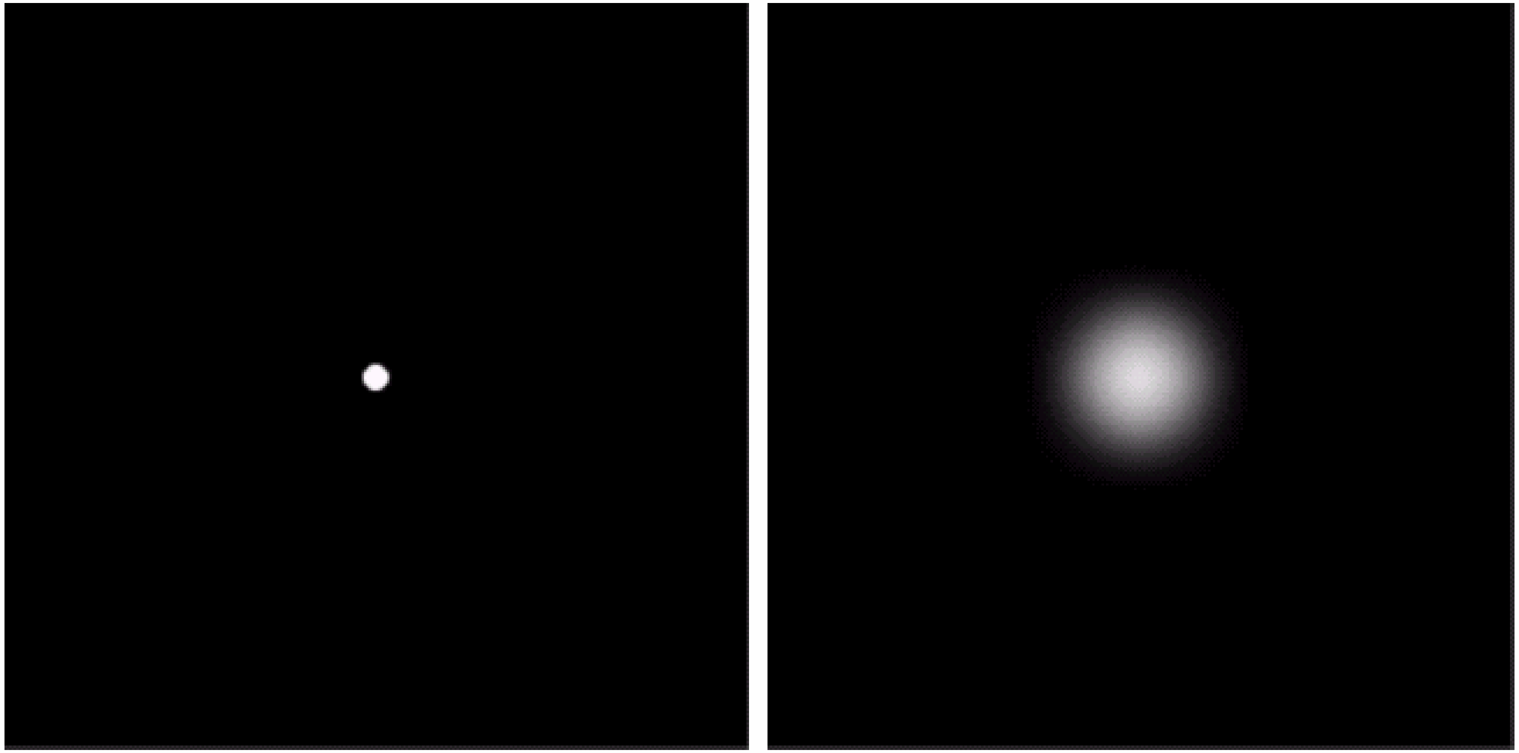
- Suppose that an image components of motion in the x- and y-directions, respectively. If T is the duration of the exposure, it follows that

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right]dt$$

$$G(u,v) = F(u,v)\int_0^T e^{-j2\pi\left[ux_0(t)+vy_0(t)\right]}dt$$

$$H(u,v) = \int_0^T e^{-j2\pi\left[ux_0(t)+vy_0(t)\right]}dt$$

$$G(u,v) = H(u,v)F(u,v)$$



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

a b
c d

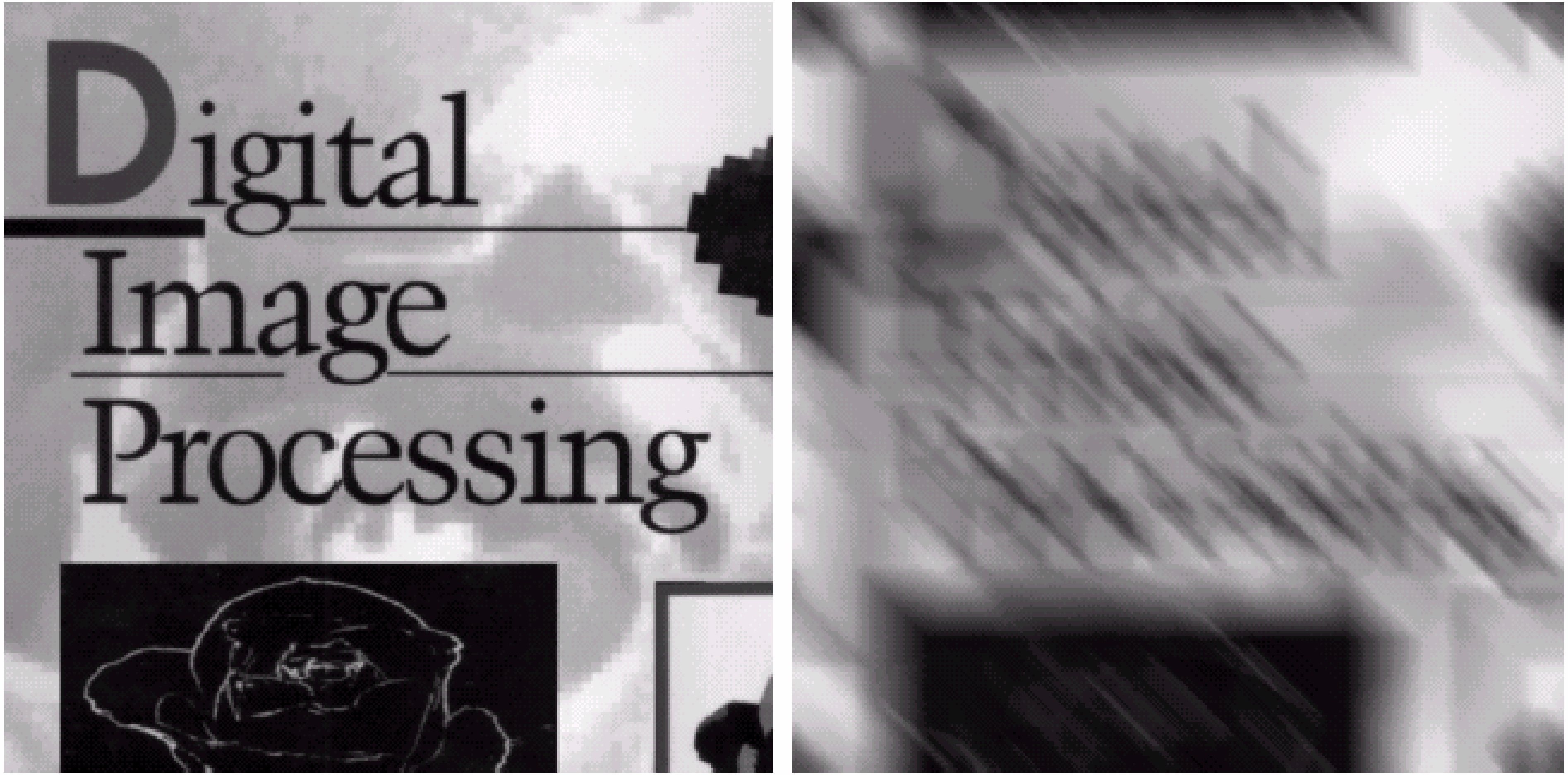
FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

- At a rate given by $x_0(t) = at/T$, $y_0(t) = 0$.

$$H(u,v) = \frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

- With the motion given by $y_0(t) = bt/T$, then the degradation function becomes

$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin\left[\pi(ua+vb)\right]e^{-j\pi(ua+vb)}$$



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

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$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

- 代入 $G(u,v) = H(u,v)F(u,v) + N(u,v)$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

a	b
c	d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

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$$e^2 = E\left\{\left(f - \hat{f}\right)^2\right\}$$

- Based on these conditions, the minimum of the error function is given in the frequency domain by the expression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v) S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)} \right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$H(u, v)$ = degradation function

$H^*(u, v)$ = complex conjugate of $H(u, v)$

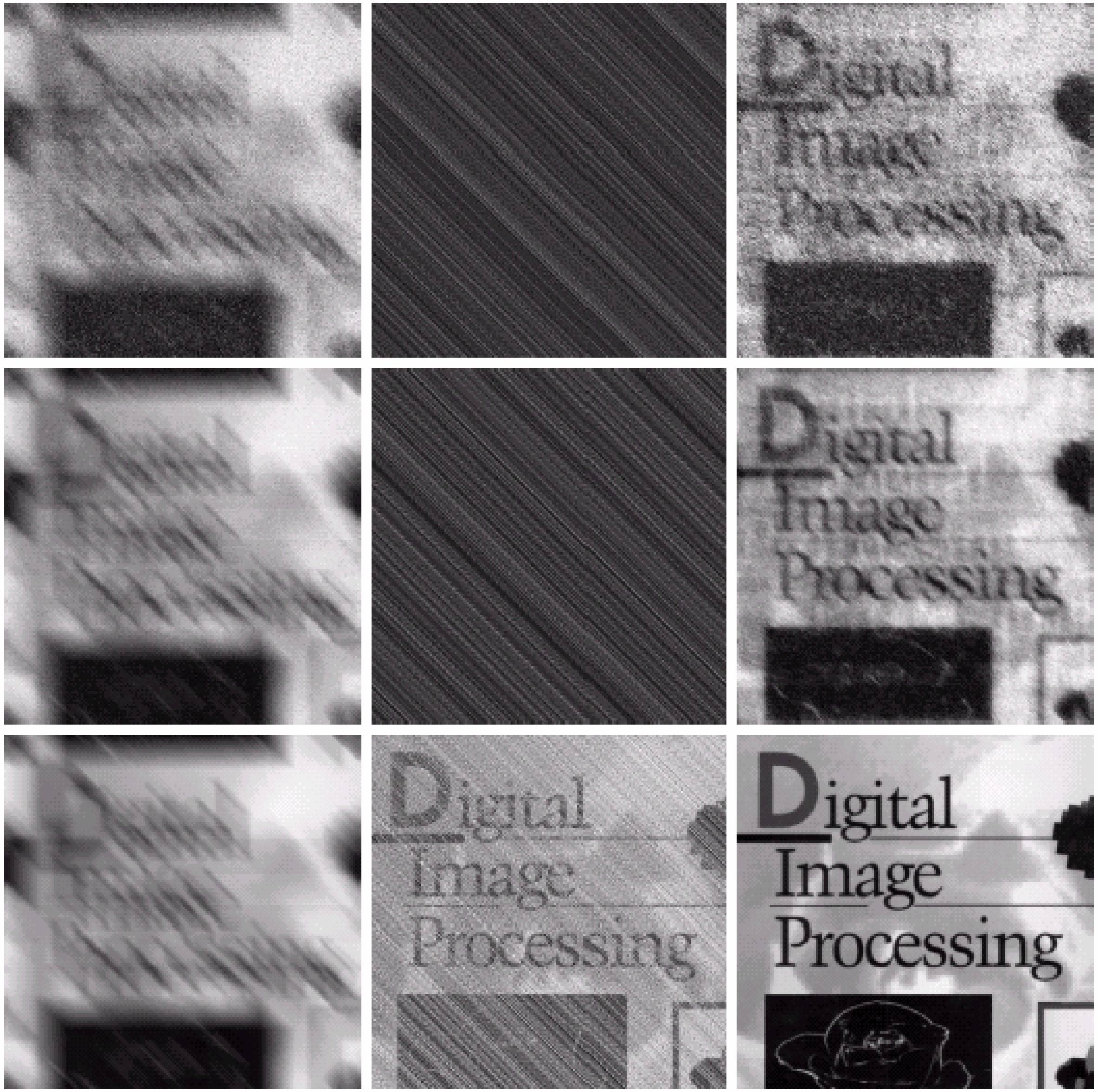
$|H(u, v)|^2 = H^*(u, v)H(u, v)$

$S_\eta(u, v) = |N(u, v)|^2$ = power spectrum of the noise [see Eq. (4.2-20)]

$S_f(u, v) = |F(u, v)|^2$ = power spectrum of the undegraded image.

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.



a	b	c
d	e	f
g	h	i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

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- The Wiener filter presents an additional difficulty:

  The power spectra of the undergraded image and noise must be known.

$$g = Hf + \eta$$

- To find the minimum of a criterion function, C, defined as

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y) \right]^2$$

Subject to the constraint

$$\left\| g - H\hat{f} \right\|^2 = \left\| \eta \right\|^2$$

- The frequency domain solution to this optimization problem is given by the expression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

- P(u,v) is the Fourier transform of the function

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

- Define a “residual” vector r as

$$r = g - H\hat{f}$$

$$\phi(\gamma) = r^T r = \|r\|^2$$

$$\|r\|^2 = \|\eta\|^2 \pm a$$

- In order to use this algorithm, we need the quantities $\|r\|^2$ and $\|\eta\|^2$.

$$R(u,v) = G(u,v) - H(u,v)\hat{F}(u,v)$$

$$\|r\|^2 = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} r^2(x,y)$$

$$\sigma_\eta^2 = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\eta(x,y) - m_\eta\right]^2$$

$$m_\eta = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\eta(x,y)$$

$$\|\eta\|^2 = MN\left[\sigma_\eta^2 + m_\eta^2\right]$$

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

a b

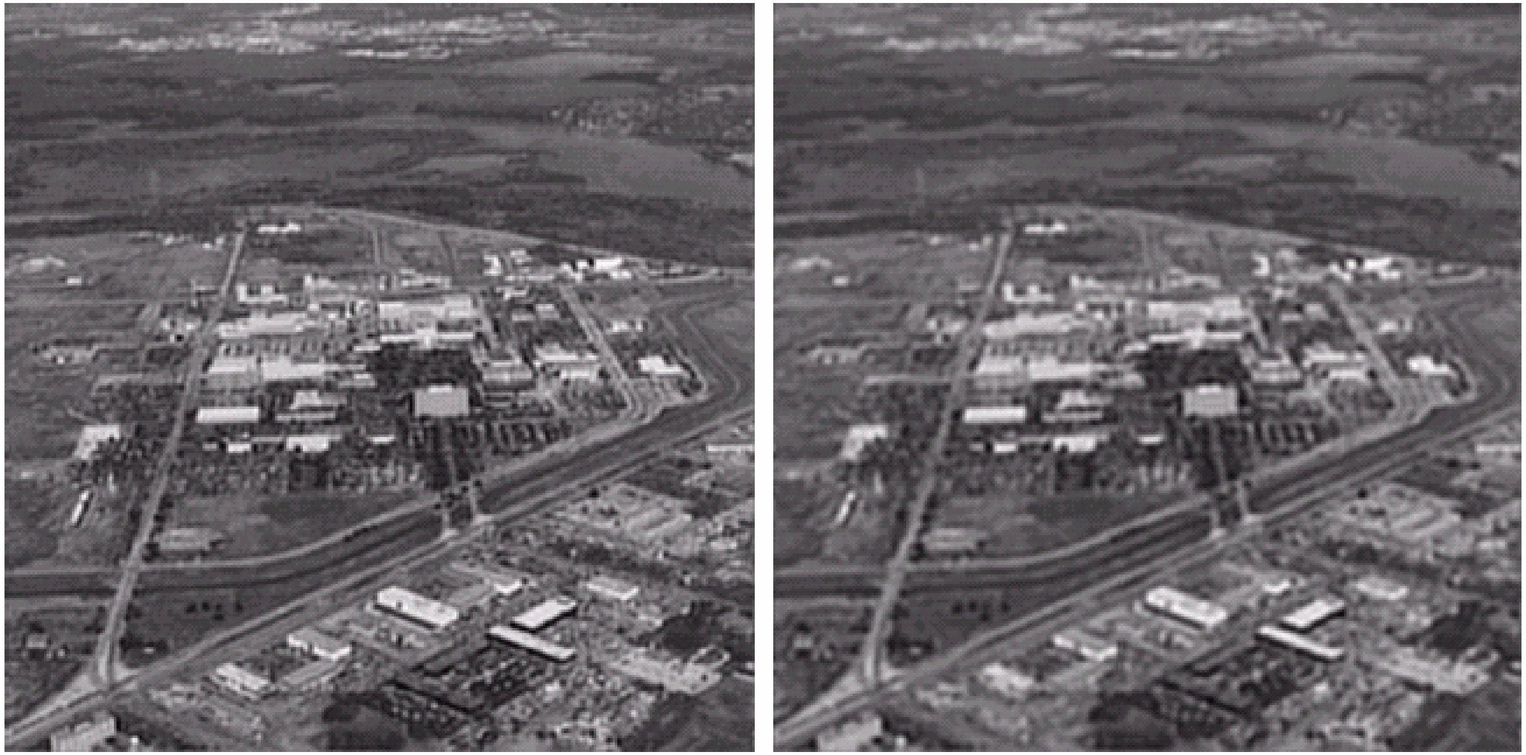

FIGURE 5.31
(a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters.
(b) Result obtained with wrong noise parameters.

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$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha}\left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha}$$

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- A spatial transformation: which defines the “rearrangement” of pixels on the image plane;
- Gray-level interpolation: which deals with the assignment of gray levels to pixels in the spatially transformed image.

Spatial Transformations

$$x' = r(x, y)$$
$$y' = s(x, y)$$

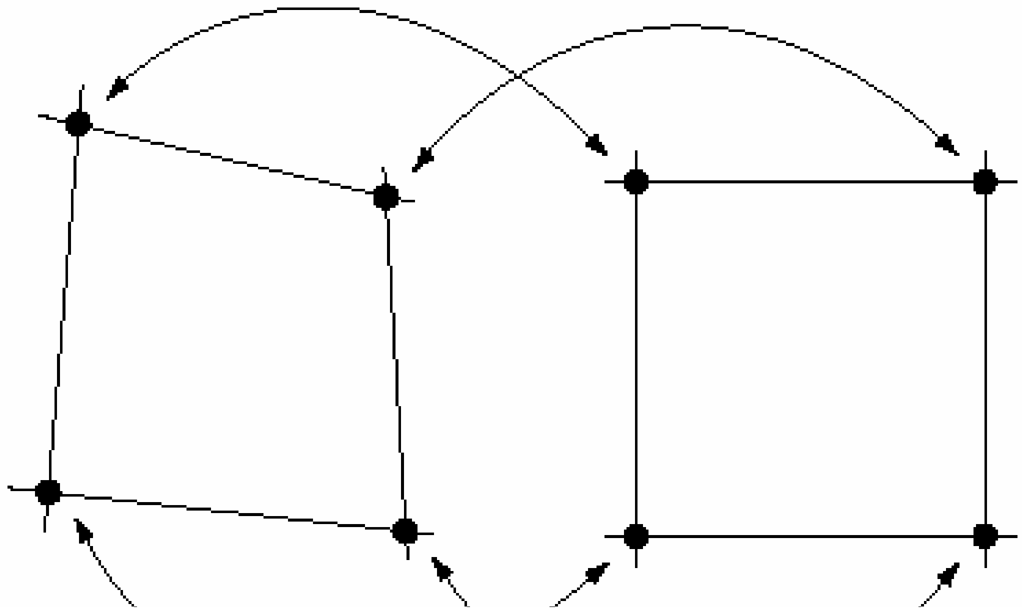


FIGURE 5.32 Corresponding tiepoints in two image segments.

$$r(x,y) = c_1 x + c_2 y + c_3 xy + c_4$$

$$s(x,y) = c_5 x + c_6 y + c_7 xy + c_8$$

and

$$x' = c_1 x + c_2 y + c_3 xy + c_4$$

$$y' = c_5 x + c_6 y + c_7 xy + c_8$$

Gray-Level Interpolation

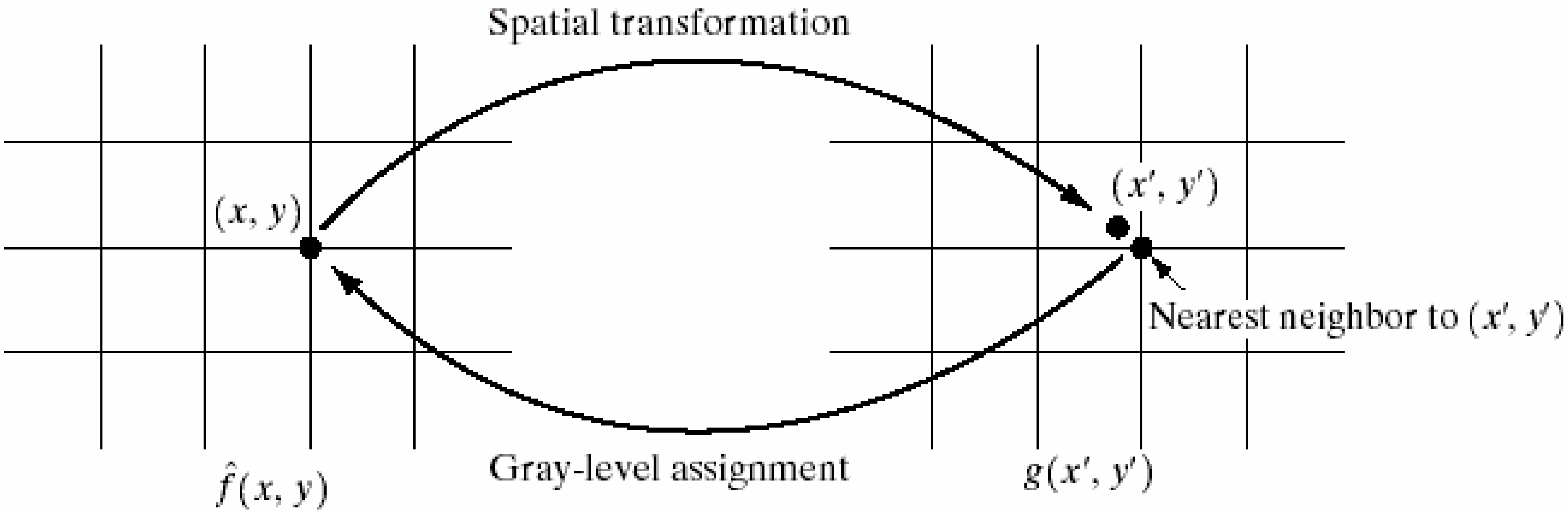
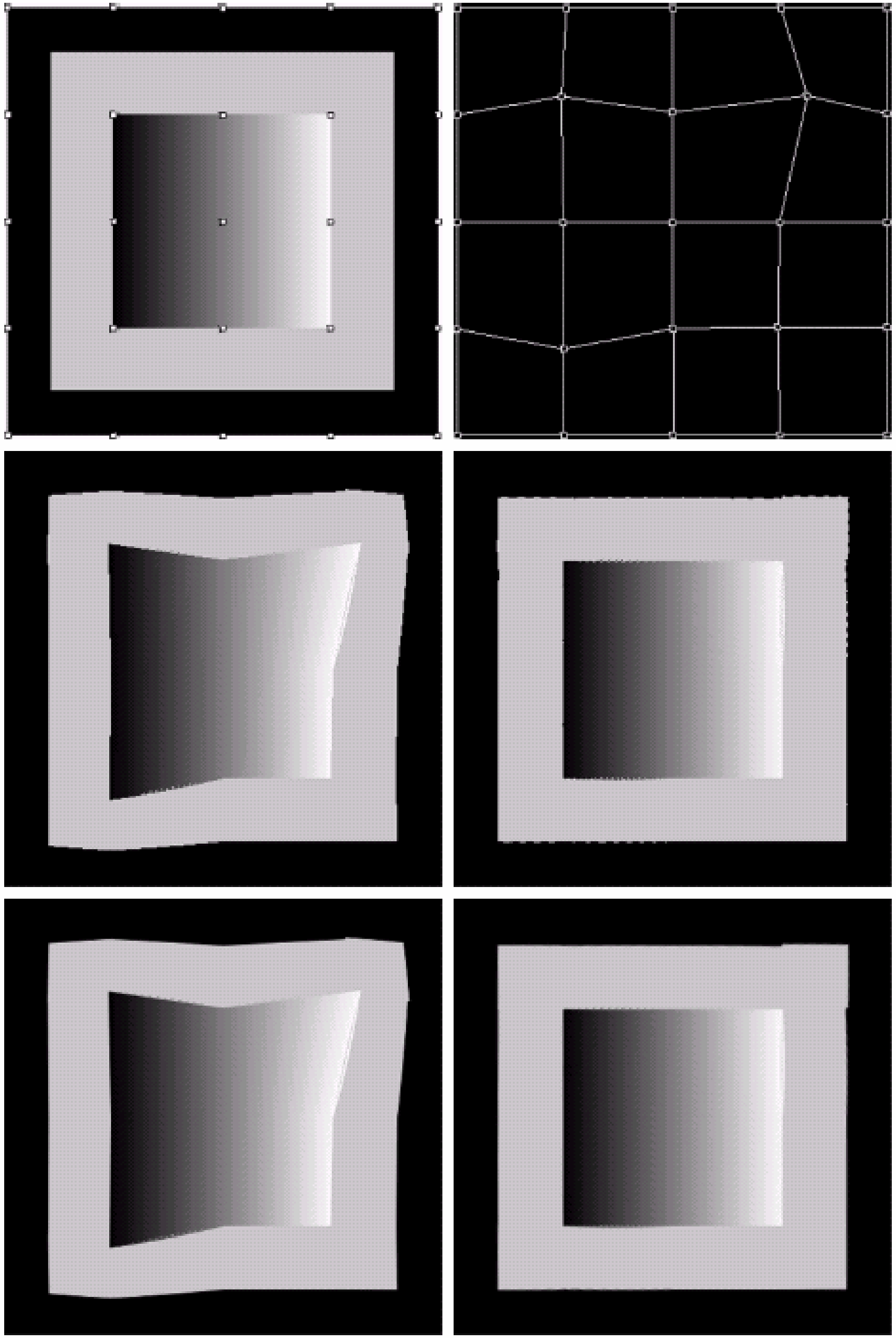


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.



a b
c d
e f

FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

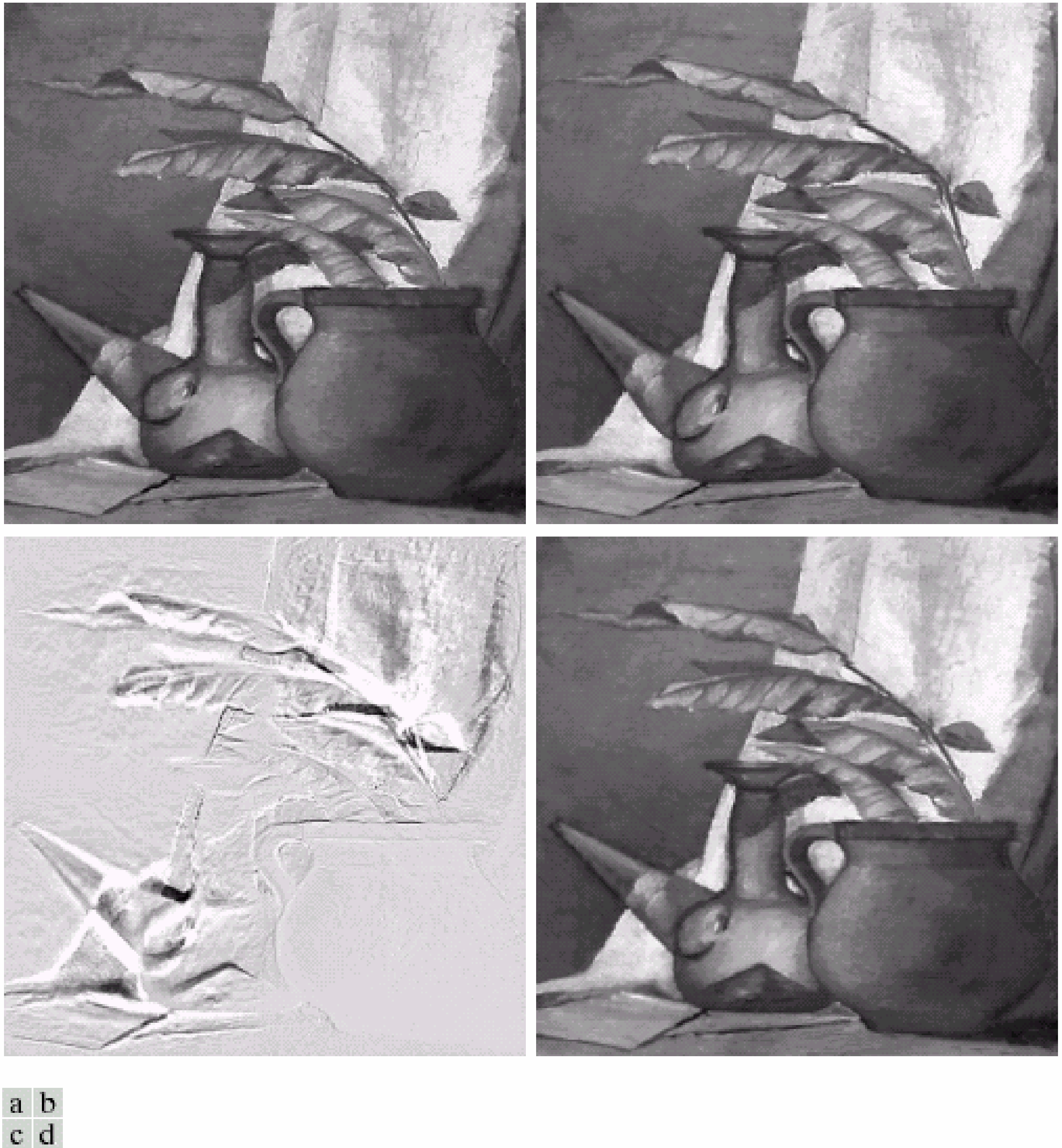


FIGURE 5.35 (a) An image before geometric distortion. (b) Image geometrically distorted using the same parameters as in Fig. 5.34(e). (c) Difference between (a) and (b). (d) Geometrically restored image.