CoE4TN4
Image Processing

Chapter 9
Morphological Image Processing

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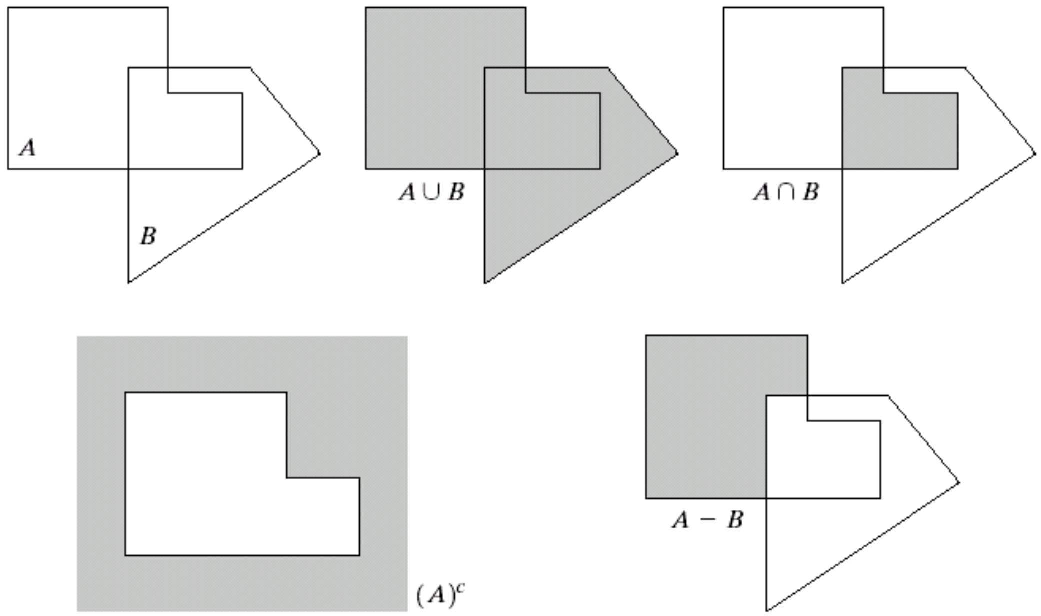
Image Morphology

- Morphology: a branch of biology that deals with the form and structure of animals and plants
- A tool for extracting image components that are useful in representation and description
- Language of morphology: set theory
- Objects in an image are represented by a sets
- For binary images each element of the set is a 2-D vector with the (x,y) coordinates of a black (or white depending on the object) pixel

Preliminaries



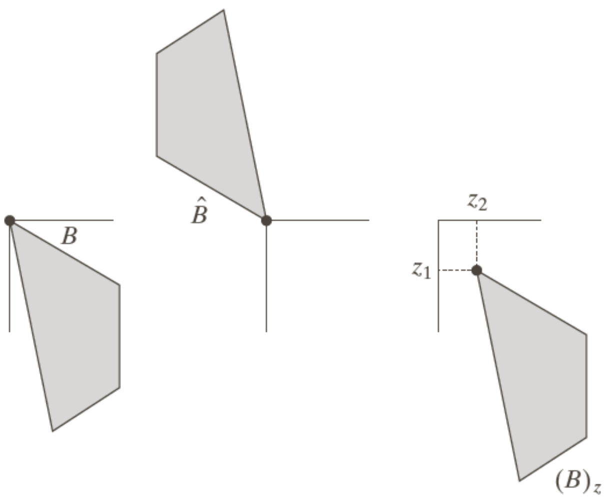
Preliminaries

- Reflection of a set:

$$\hat{B} = \{w | w = -b, \text{for} \;\; b \in B\}$$

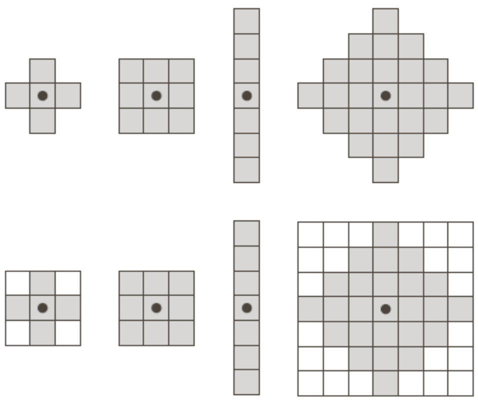
- Translation:

$$(B)_z = \{c | c = b + z, \text{for} \;\; b \in B\}$$



Preliminaries

- Structuring element: small image (set) used to probe the image under study
- Center of structuring element is typically marked
- For a computer implementation, SE should be rectangular array
- This is done by appending smallest number of background elements to the SE to form a rectangle



Erosion

- Erosion:

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

- Erosion of A by B is the set of all displacements z such that B translated by z is contained in A
- B is the structuring element
- One of the applications of erosion is elimination of irrelevant details

Erosion

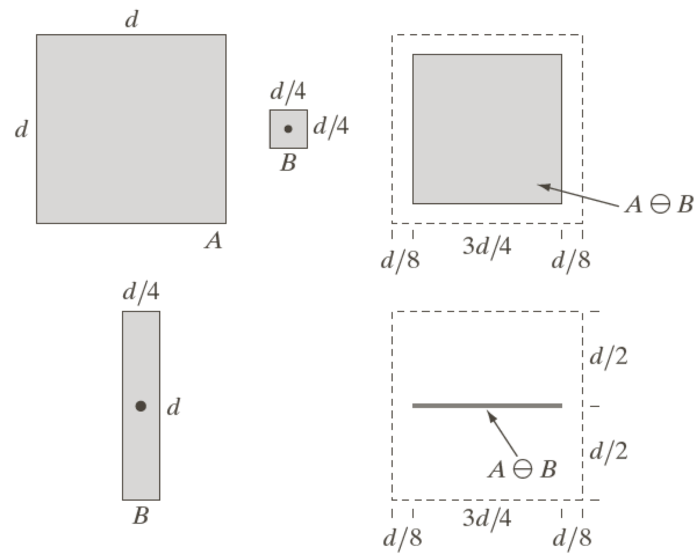
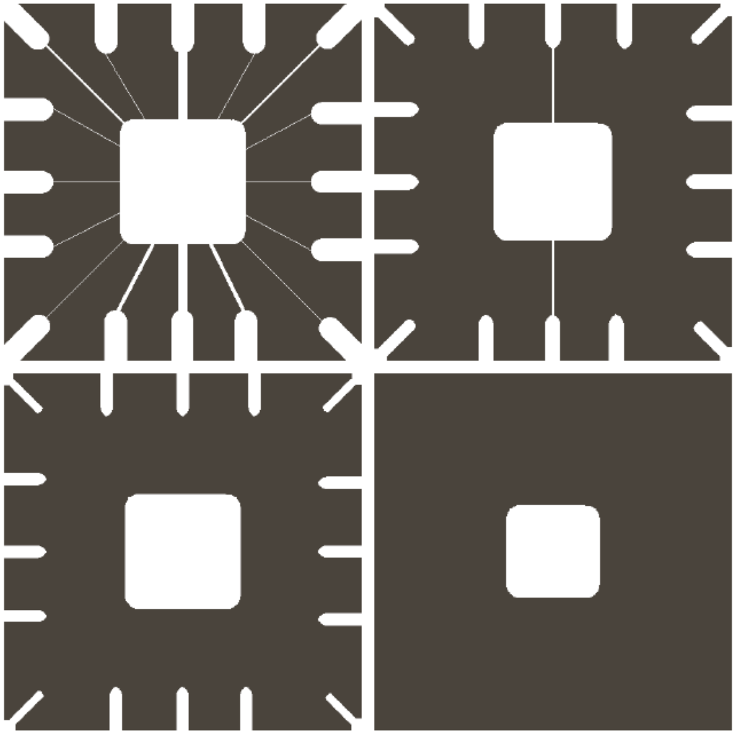


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion



a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

- Dilation:

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

- Obtain the reflection of B and shift it by z.
- Dilation of A by B is the set of all displacements z such that $B^{\wedge}$ and A overlap at least one element
- B is the structuring element
- One of the applications of dilation is for bridging gaps

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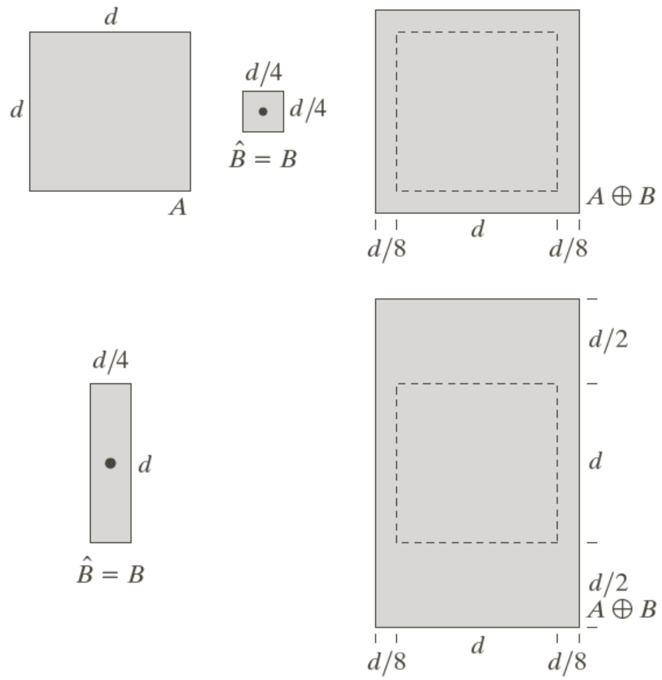
Dilation



a b c
d e

FIGURE 9.6
(a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

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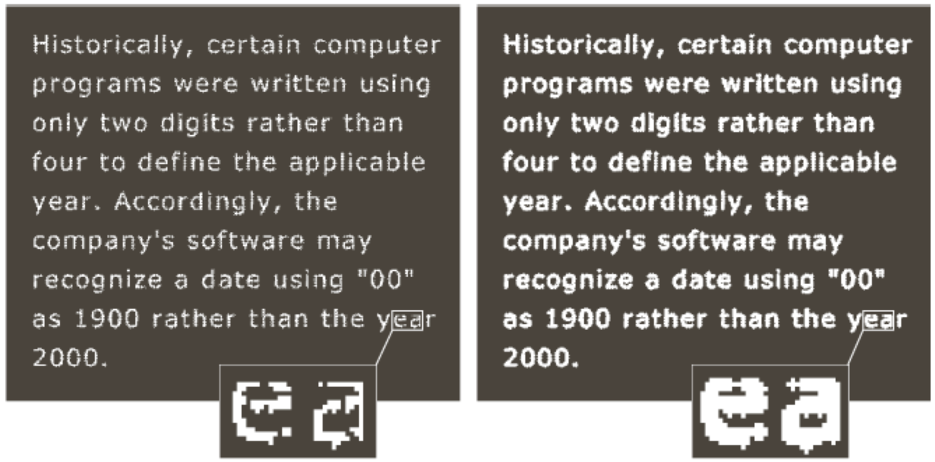
Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

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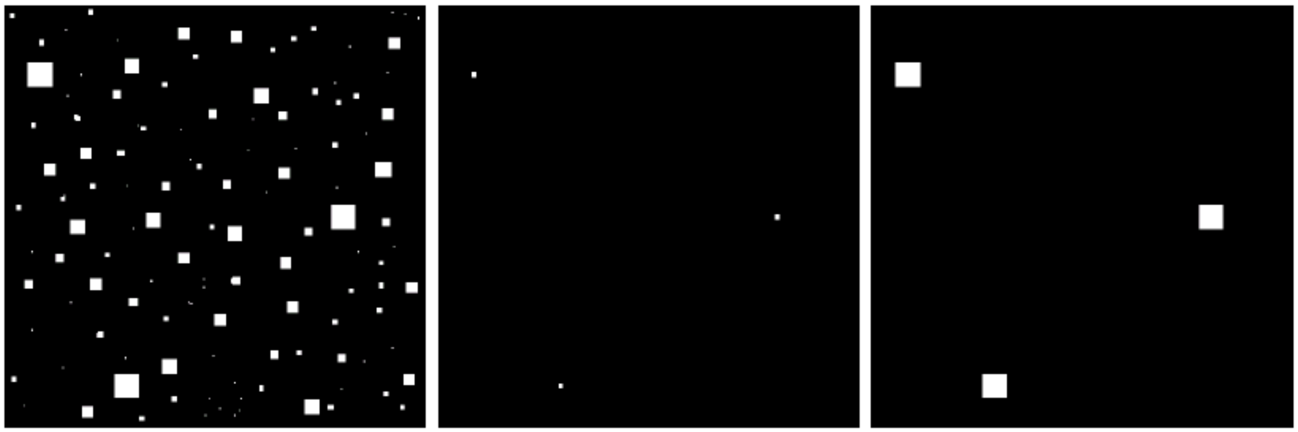
Dilation & Erosion



Opening and closing

- Opening: smoothes the contour of an object, breaks narrow strips and eliminates thin protrusions (bulges)
- Opening of set A by structuring element B is defined as:

$$A \circ B = (A \ominus B) \oplus B$$

- Closing of A by structuring element B is defined as:

$$A \bullet B = (A \oplus B) \ominus B$$

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Opening and closing

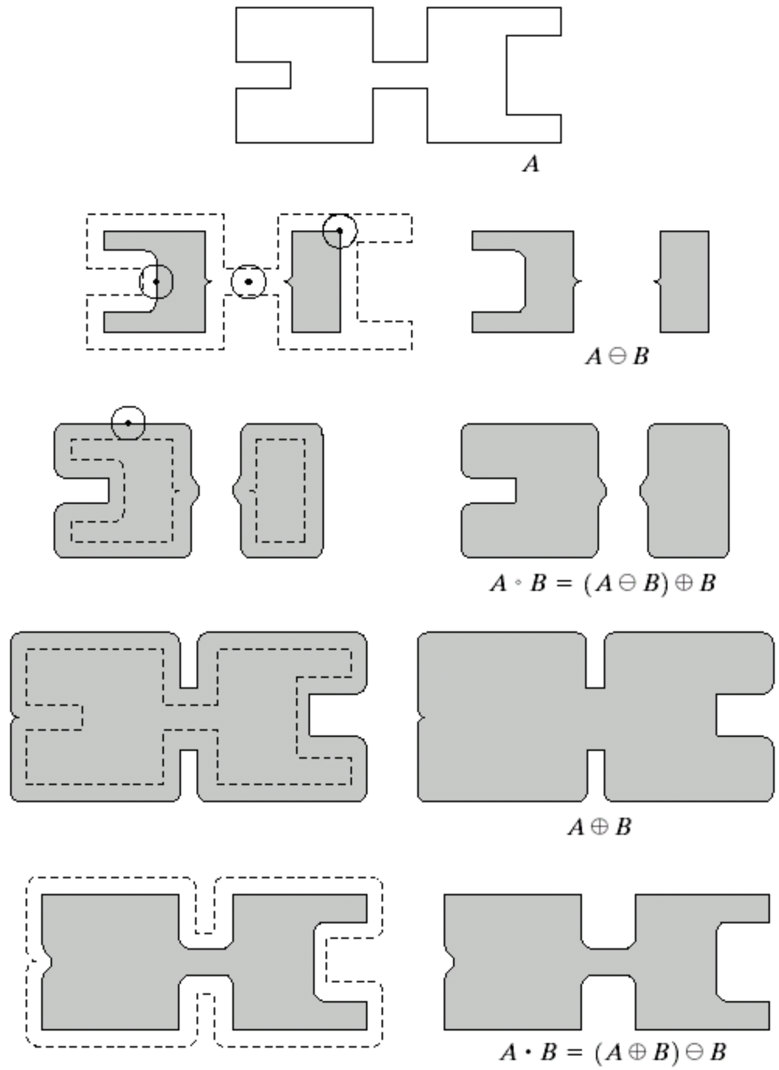

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

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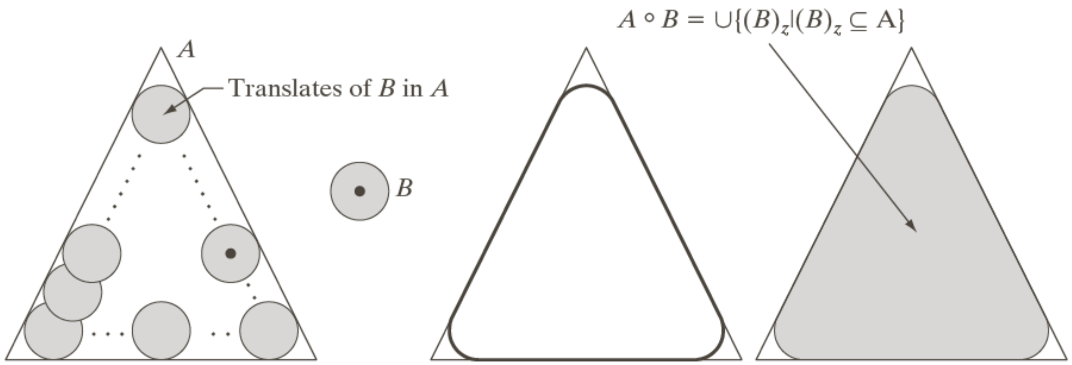
Opening

- The effect of opening can be quite easily visualized. Imagine taking the structuring element and sliding it around inside foreground region, without changing its orientation. All pixels which can be covered by the structuring element with the structuring element being entirely within the foreground region will be preserved. However, all foreground pixels which cannot be reached by the structuring element without parts of it moving out of the foreground region will be eroded away.

Opening

- After the opening has been carried out, the new boundaries of foreground regions will all be such that the structuring element fits inside them, and so further openings with the same element have no effect.
- The property is known as idempotence.

Opening



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

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Closing

- The effect of closing can be quite easily visualized. Imagine taking the structuring element and sliding it around outside each foreground region, without changing its orientation. For any background boundary point, if the structuring element can be made to touch that point, without any part of the element being inside a foreground region, then that point remains background. If this is not possible, then the pixel is set to foreground.

Closing

- After the closing has been carried out the background region will be such that the structuring element can be made to cover any point in the background without any part of it also covering a foreground point, and so further closings will have no effect.
- This property is known as idempotence.

Closing

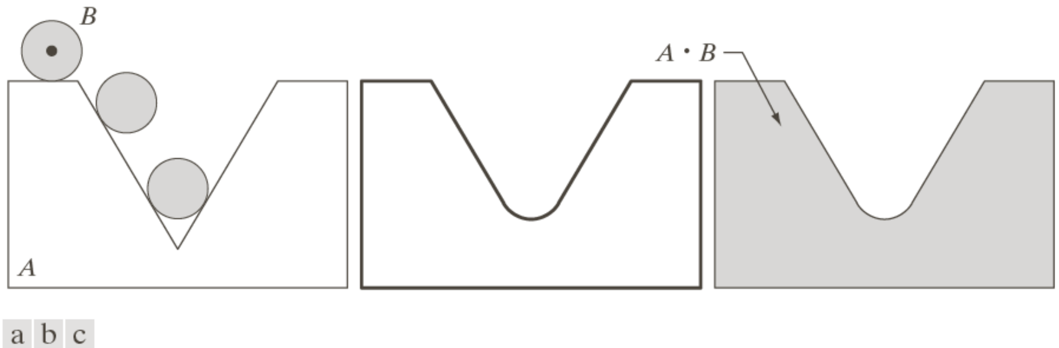

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

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Hit-or-Miss transform

- Hit-or-miss transform is a basic tool for shape detection
- Set A consists of three shapes: X, Y, Z
- The objective is to find the location of X
- Let X be enclosed by a small window W
- Local background of X with respect to W is W-X
- Set of locations for which X exactly fits inside A is the intersection of the erosion of A by X and the erosion of A^c by (W-X)

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

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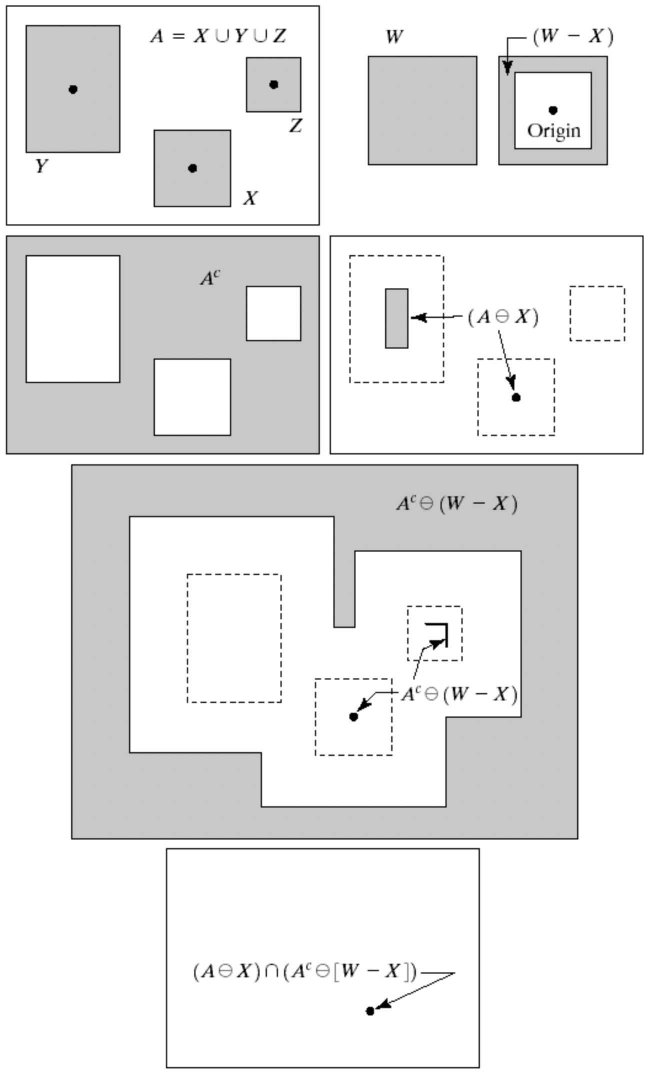
Hit-or-Miss transform



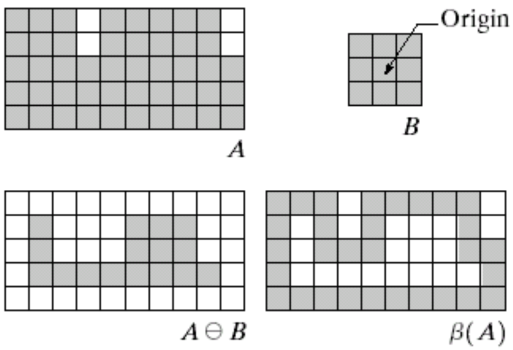
Boundary Extraction

- Boundary of a set A, denoted by β(A) can be obtained by first eroding A by B and then performing the set difference between A and its erosion:

$$\beta(A) = A - (A \ominus B)$$

a	b
c	d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*. (d) Boundary, given by the set difference between *A* and its erosion.



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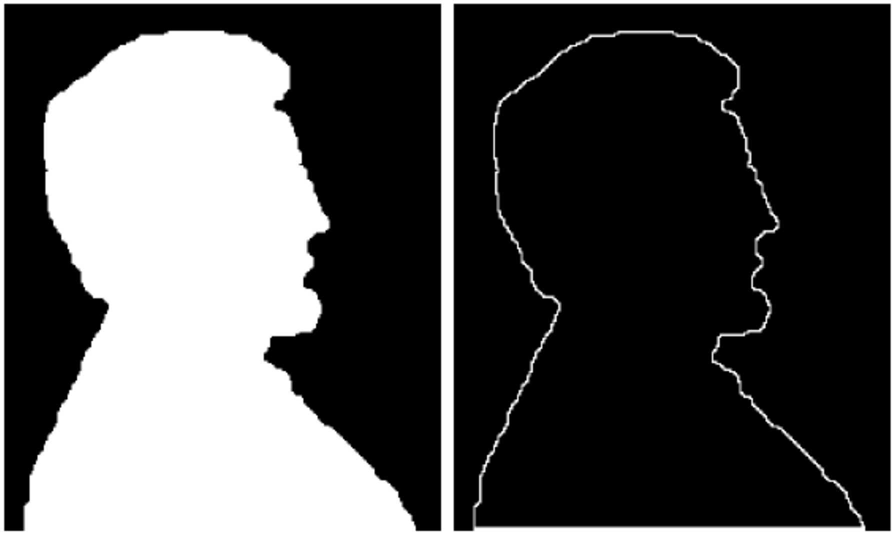
Boundary



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region filling

- Let's A denote the boundary points of a region and p is a point inside the boundary
- Objective is to fill the entire region with 1s.

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

X_0=p

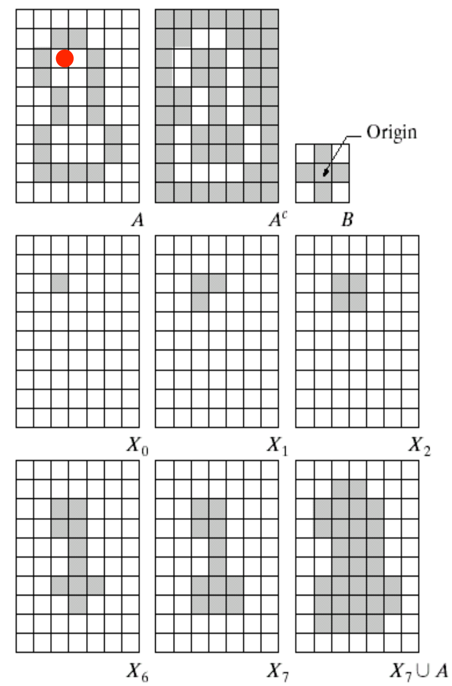
B is the structuring element

Algorithm terminates if X_k=X_{k-1}

Union of X_k and A is the region filled

a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set *A*. (b) Complement of *A*. (c) Structuring element *B*. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



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Extraction of connected components

- Extraction of connected components in a binary image is central to many automated image analysis applications
- Let Y represent a connected component contained in a set A and assume that a point p of Y is known. Then the following iterative expression yields all the elements of Y:

$$X_k = (X_{k-1} \oplus B) \cap A$$

X_0=p

B is the structuring element

Algorithm terminates if X_k=X_{k-1}

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Extraction of connected components

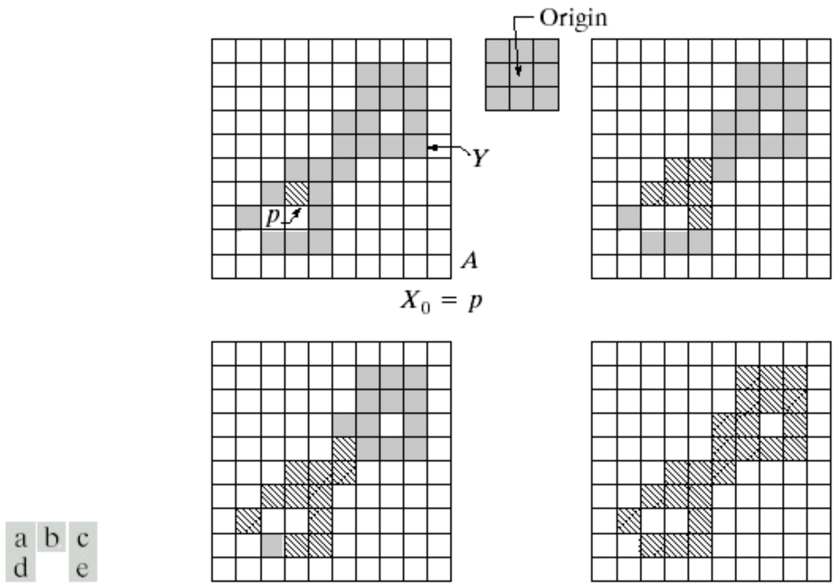

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Convex hull

- A set *A* is said to be *convex* if the straight line joining any two points in *A* lies entirely within *A*.
- The *convex hull H* of a set *S* is the smallest convex set containing *S*.
  - The set *H-S* is called the convex difference, which is useful for object description.
- The procedure is to implement the equation:

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$$

  - With $X^i_0 = A$. Let $D^i = X^i_k$, when $X^i_k = X^i_{k-1}$. The convex hull of *A* is

$$C(A) = \cup_{i=1}^4 D^i$$

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Convex hull

- The procedure for obtaining convex hull consists of iteratively applying the hit-or-miss transform to A with B^1 (B^i, i = 1, 2, 3, 4, represent four structuring elements.); when no further changes occur, we perform the union with A and call the result D1. The procedure is repeated with B2 until no further changes occur, and so on.
- The union of the four resulting D's constitutes the convex hull of A.
- The structuring element used in convex hull is a slight extension to the type that has been introduced for erosion and dilation, it can contain both both ones and zeros (and even don't cares)

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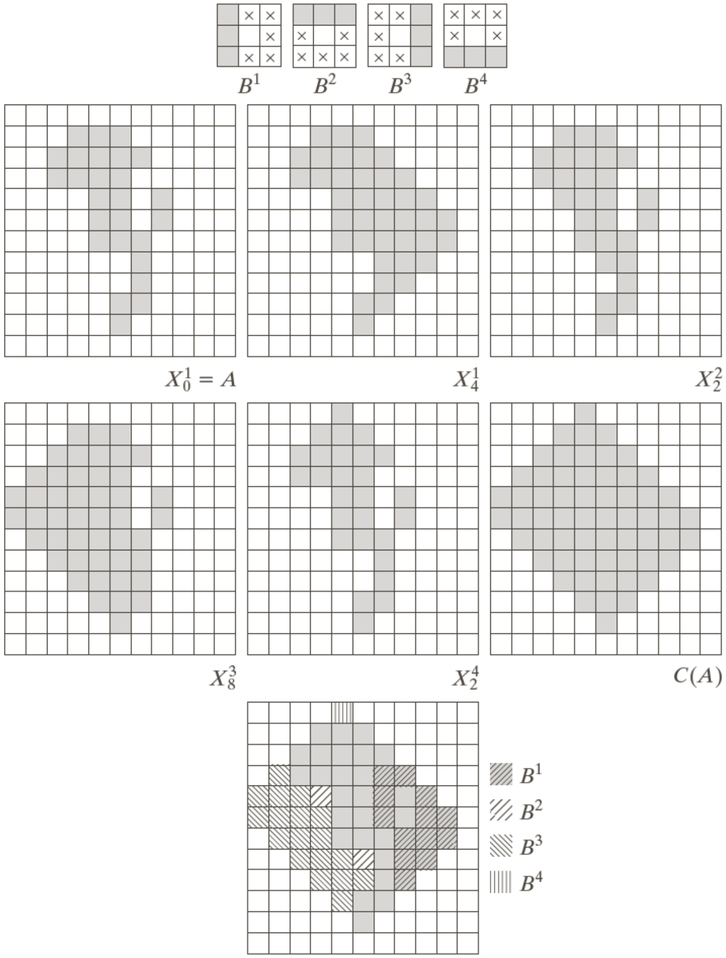
Convex hull



a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

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Thinning

- The thinning of a set *A* by a structuring element *B*, denoted *A* *B*, is defined by

$$A \otimes B = A - (A \circledast B)$$

- Each *B* is usually a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, \ldots, B^n\}$$

  - B^1, B^2,…are different rotated versions of *B*.

$$A \otimes \{B\} = (\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

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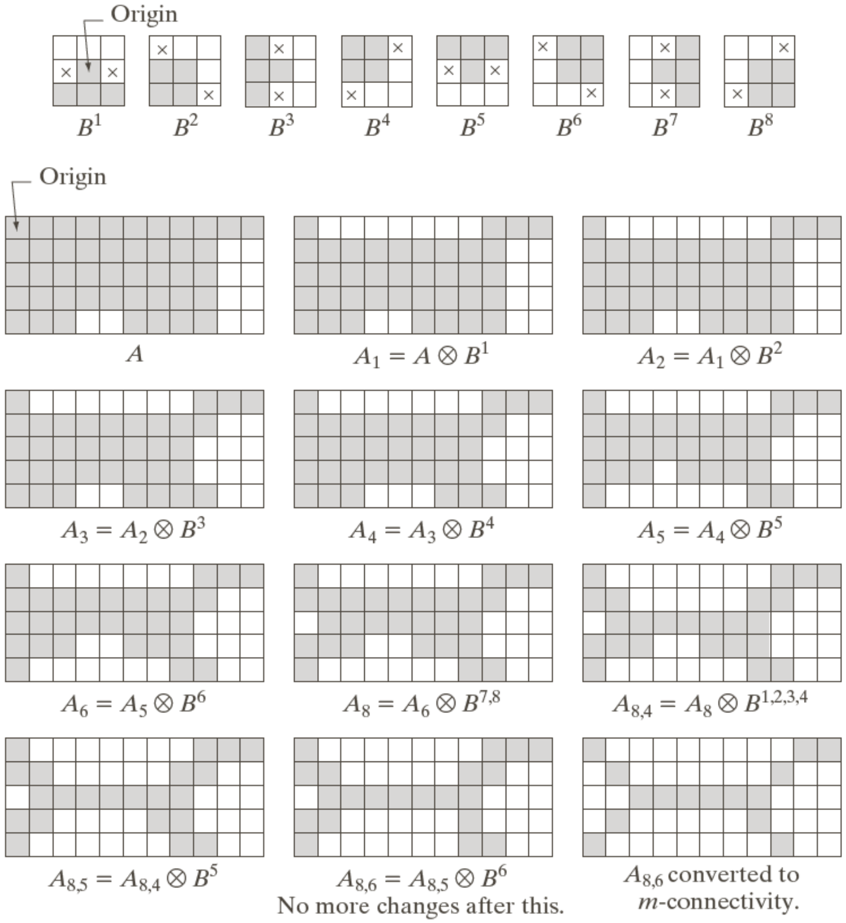
Thinning



a		
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

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Thickening

- Thickening is dual of thinning and is defined by

$$A \odot \{B\} = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

- A more efficient scheme is to obtain the complement of A, C=A^c, thin C and then to compute C^c.

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Thickening

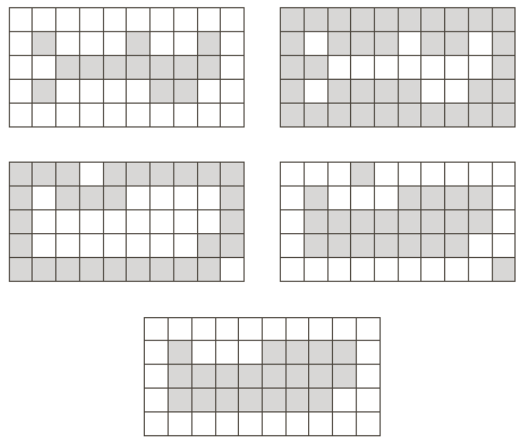
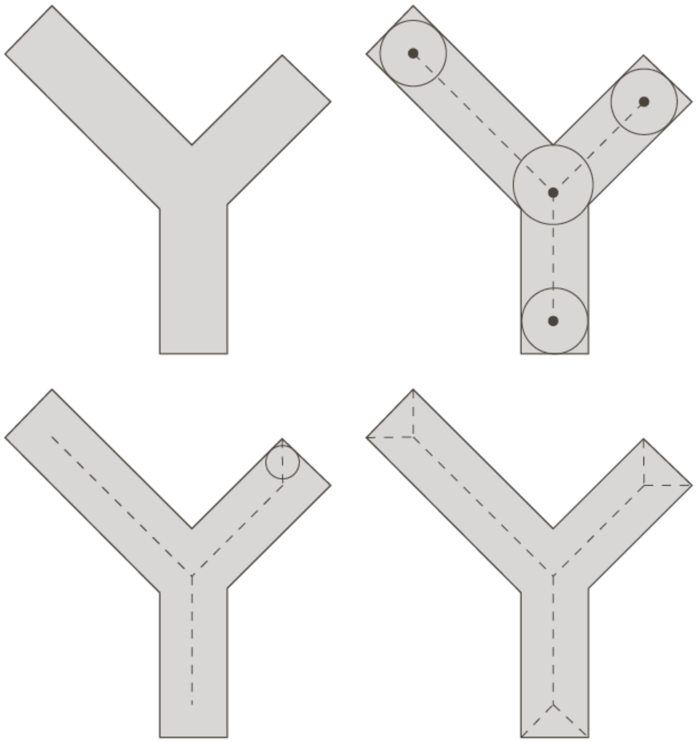


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons



a b
c d

FIGURE 9.23
(a) Set *A*.
(b) Various positions of maximum disks with centers on the skeleton of *A*.
(c) Another maximum disk on a different segment of the skeleton of *A*.
(d) Complete skeleton.

Skeletons

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = ((\ldots((A \ominus B) \ominus B)\ldots) \ominus B)$$

$$K = max\{k|(A \ominus kB) \neq \emptyset\}$$

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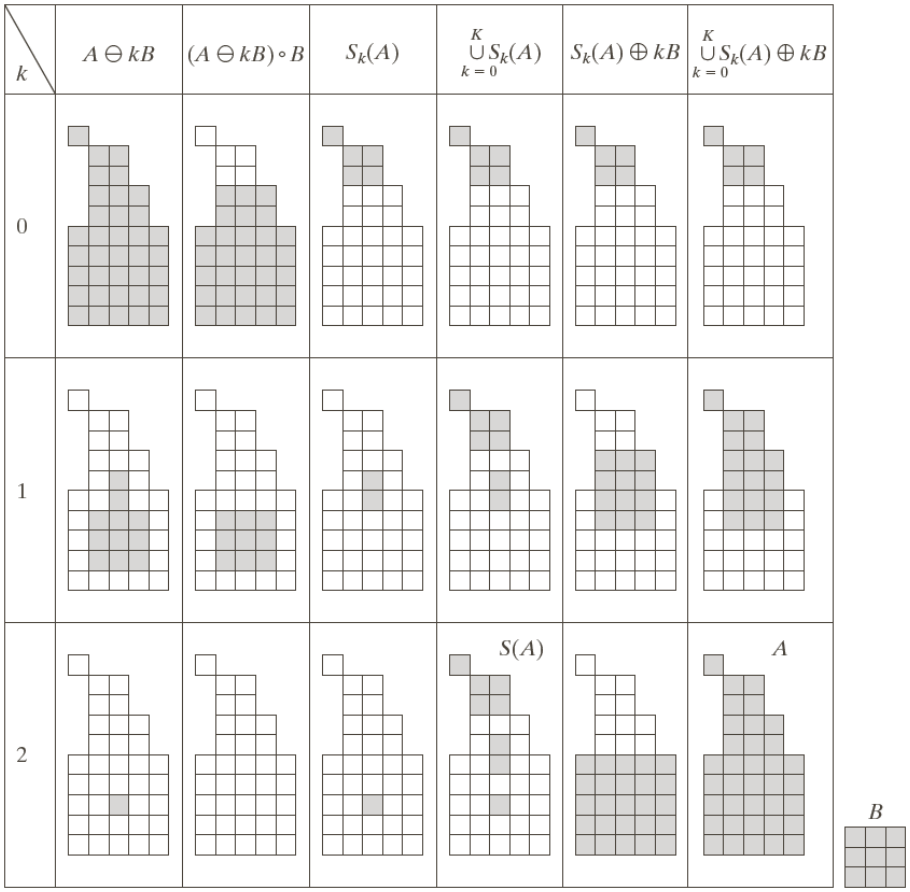
Skeletons



k
$A \ominus kB$
$(A \ominus kB) \circ B$
$S_k(A)$
$\bigcup_{k=0}^{K} S_k(A)$
$S_k(A) \oplus kB$
$\bigcup_{k=0}^{K} S_k(A) \oplus kB$
0
1
2
S(A)
A
B

Pruning

- Clean up the parasitic components of the set
  - Complement step of thinning and skeletons
- Application: hand-printed character recognition
- Find skeleton of each character
- Produce spurs during erosion by non-uniformities in the strokes

Pruning

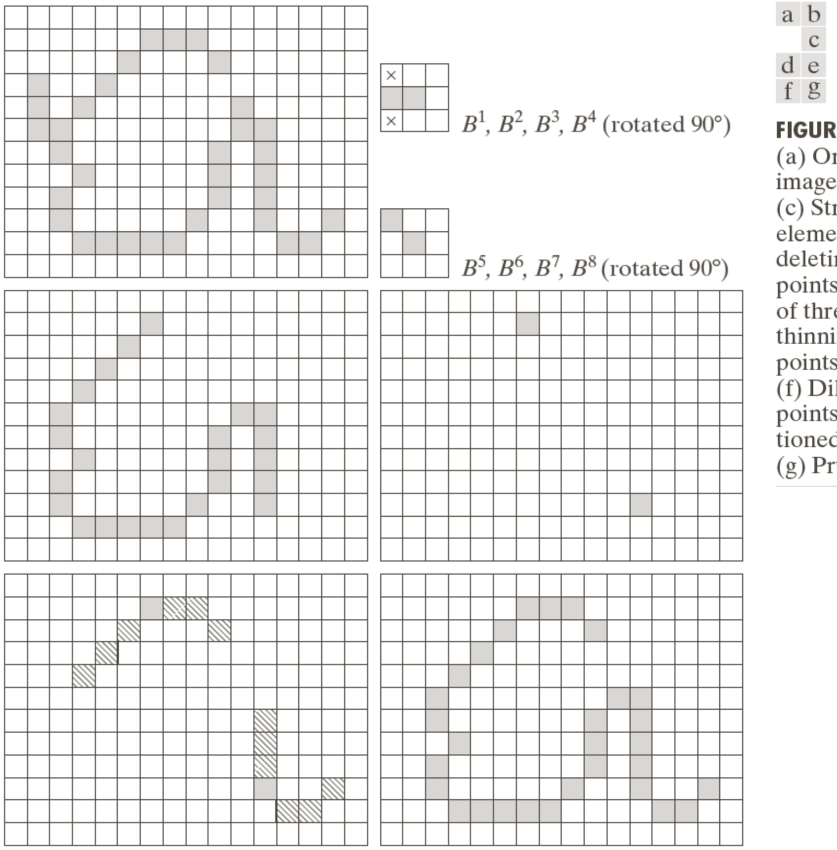


FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

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